Supplementary Material to Resolving Policy Conflicts in Multi-Carrier Cellular Access

Proofs to the theoretical results in paper: Resolving Policy Conflicts in Multi-Carrier Cellular Access

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In this document, we provide proofs for the theoretical results in the paper [1].

We reiterate the notations (Table 2 in the full paper) for the ease of referring them in the proofs here.

<table>
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<th>Notation</th>
<th>Description</th>
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<td>$C_i$</td>
<td>Carrier $i$, $i \in [1, N]$</td>
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<tr>
<td>$RAT_j$</td>
<td>Radio access technology $j$ (e.g. 3G, 4G)</td>
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<td>$c^k/c_i^k$</td>
<td>Cell $k$ (in carrier $C_i$)</td>
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<td>$P_{i,j}/P_i$</td>
<td>Inter-carrier preference on carrier $C_i$’s $RAT_j/C_i$</td>
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<td>$p(c^j)$</td>
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<td>$M, M(C_i)$</td>
<td>Measure $M$ (on $C_i$) for inter-carrier policy</td>
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<td>$\delta, \theta, \phi$</td>
<td>Different inter-carrier thresholds (on carrier)</td>
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A PROOFS OF THEOREMS FOR PREFERENCE-BASED POLICY

A.1 Proof of Proposition 1

Proof. Following the static condition assumption, neither inter-carrier policy nor intra-carrier policy changes. Therefore the decision will be the same under deterministic policy, and loop is persistent by definition. □
A.2 Proof of Lemma 1

Proof. (Sufficiency $\Rightarrow$) The sequence $(\ast) C_1 \mapsto C_2 \mapsto \cdots \mapsto C_N \mapsto C_1$ by definition is an $N$-carrier loop.

(Necessity $\Leftarrow$) We show in three steps that, if an inter-carrier switching sequence contains an $N$-carrier loop, then it contains the sequence $(\ast) C_1 \mapsto C_2 \mapsto \cdots \mapsto C_N \mapsto C_1$. Suppose the phone initially connects to carrier $C_0$'s $RAT_0$. We denote the highest preference (carrier, RAT) combination in carrier $C_i$ as $P_i^\text{max} = \max_j P_{i,j}$.

Step 1. We show that under any initial condition ($C_0$ that phone is connected to), the device will be served by $C_1$ after finite switching steps. We prove it by cases. If $C_0 = C_1$, then the conclusion holds. If $C_0 \neq C_1$, then an inter-carrier switching $C_0 \mapsto C_1$ occurs. This is because $P_i^\text{max} \geq P_0^\text{max} \geq P_{0,0}$ and $C_1$ is the carrier with the smallest index. According to Policy 1, such switch will happen. Therefore, the device will always be served by $C_1$ initially or after finite steps.

Step 2. The inter-carrier switching from $C_1 \mapsto C_2$ must occur, given that an $N$-carrier loop exists. We can prove it by contradiction: If $C_1 \mapsto C_2$ does not happen, there are two possibilities: either (a) the inter-carrier logic decides to not switch at $C_1$, or that (b) $C_1 \mapsto C_i, i \neq 2$ occurs. For case (a), the conditions are that (1) $C_1$ has the highest preference and (2) $C_1$ is available. Such case does not hold, because though the first condition ($C_1$ has the highest preference) holds by assumption, the second condition does not hold. If it is true, then there is no other possibilities nor reason to switch out from $C_1$, therefore the $N$-carrier loop will not exist. For case (b), this does not hold because $P_i^\text{max} \geq P_0^\text{max} \geq P_{0,0}, j \in [3, N]$. Therefore if a switching out from $C_1$ happens, it will switch to $C_2$ according to Policy 1, not any other carrier $C_i, i \neq 2$. Therefore, it must be the case that the inter-carrier switching $C_1 \mapsto C_2$ happens.

Step 3. Similar to the above proof, subsequent inter-carrier switchings $C_2 \mapsto C_3, C_3 \mapsto C_4, \ldots, C_N \mapsto C_1$ must occur in order and no other switching sequences may occur, otherwise the inter-carrier switching sequence will stop at any of the carriers $C_3, C_4, \ldots, C_N$ but no $N$-carrier loop exists. \hfill $\Box$

A.3 Proof of Theorem 6.1

Proof. (Sufficiency $\Rightarrow$) Suppose both conditions satisfy. Without loss of generality, suppose $RAT_1$ is $RAT_H$ and $RAT_2$ is $RAT_L$. This further implies: (i) for the condition (a) stated in Theorem 6.1, $P_{\text{max}} = P_{1,1} \geq P_{i,j}, \forall i \in [1, N], \forall j \in [3, N]$, and that $RAT_2$'s preference is always lower than $RAT_1$: $P_{i,2} < P_{1,1}, \forall i \in [1, N]$. (ii) for the condition (b), the intra-carrier logic in every carrier will prefer $RAT_2$: as long as the phone is not connected to $RAT_2$, intra-carrier logic will move phone to $RAT_2$. We next constructively prove that the inter-carrier switching sequence $(\ast)$ occurs.

Step 1. Starting from $C_0$ and $RAT_0$ initially, we show that phone will be connected to $C_1$ initially or in finite steps. If $C_0 = C_1$ then it is true already; otherwise, suppose $C_0 \neq C_1$ and there are two subcases: (case 1) if $RAT_0 \neq RAT_1$; according to Policy 1, since $P_{0,0} \leq P_0^{\text{max}} = P_{1,1}$, the inter-carrier switching will select $C_1, RAT_1$ and the phone will connect to $C_1$; (case 2) if $RAT_0 = RAT_1$; according to intra-carrier policy,
phone will be reselected to RAT2. Next, inter-carrier policy will select $C_1$, RAT1 and switch to $C_1$, due to the same reason as (case 1).

Step 2. We show that the inter-carrier switchings $C_i \mapsto C_{i+1}, \forall i \in [1, N - 1]$ occur. We prove it by induction.

(Base case) First, the switching $C_1 \mapsto C_2$ will occur. After Step 1, phone is connected to $C_1$. Following condition (b), $C_1$’s intra-carrier policy moves the phone from RAT1 to RAT2. However, since $P_{\text{max}} = P_{2,1} > P_{1,2}$ and that $C_2$ is unselected carrier, phone will switch to $C_2$ according to Policy 1. Moreover, $C_1$ will not switch to $C_3, C_4, \ldots, C_N$, because that all these carriers have larger index than $C_2$.

(Inductive step) Next, suppose that it is true for $k, k \in [2, N - 2]$ (which means that $C_k \mapsto C_{k+1}$ occurs), we show that it is true for $k + 1$. Since $C_k \mapsto C_{k+1}$ occurs, it means two things: (a) Inter-carrier logic chooses $C_k$, RAT1 according to Policy 1; (b) $C_1, C_2, \ldots, C_k$ have been selected, while $C_{k+1}, C_{k+2}, \ldots$ have not been selected. Given condition (b), intra-carrier logic at $C_{k+1}$ moves to RAT2. Since $P_{\text{max}} = P_{k+2,1} > P_{k+1,2}$, inter-carrier logic will perform switch. The switch target is $C_{k+2}$, because $C_1, \ldots, C_{k+1}$ have been selected so that $C_{k+2}$ is the highest preference carrier which has not been selected, and has the smallest index among all possible carriers. Together, $C_i \mapsto C_{i+1}, \forall i \in [1, N - 1]$ occur.

Step 3. We show that the inter-carrier switching $C_N \mapsto C_1$ occurs. Following Steps 1 and 2, we have selected all carriers with highest preference: $C_i, \forall i \in [1, N]$. Therefore, when the phone connects to $C_N$, all carriers are marked “unselected” again following Policy 1. When $C_N$’s intra-carrier logic moves phone from $C_N$, RAT1 to $C_N$, RAT2, an inter-carrier switching happens because $P_{\text{max}} = P_{n,1} = P_{N,1} > P_{N,2}$. Therefore, it will select $C_1$, RAT1, since it has the highest preference and $C_1$ is not selected and has the smallest index.

Together, with Steps 1, 2 and 3, we prove that the sufficient condition will lead to the inter-carrier switching sequence (1) $C_1 \mapsto C_2 \mapsto \cdots \mapsto C_N \mapsto C_1$. With the Lemma 1, the $N$-carrier loop occurs.

(Necessity $\Leftarrow$) We prove via contrapositive. The original statement is: if $N$-carrier loop happens, then both two conditions holds. We prove the contrapositive statement: if one of the conditions does not hold, $N$-carrier loop will not happen.

First, assume the condition (a) does not hold. We are proving: if some carriers have no RAT assigned with highest preference $P_{\text{max}}$, then no $N$-carrier loop may happen. It is easy to prove, because the carrier with no RAT assigned with highest preference will not get selected by inter-carrier Policy 1. Under this case, a $k$-carrier loop ($1 < k < N$) may happen, but not the $N$-carrier loop.

Second, assume the condition (b) does not hold. We are proving: if in some carriers, phone can stay in the RATH due to intra-carrier policy, then no $N$-carrier loop may happen. This is evident. When the phone stays in RATH, it satisfies both inter-carrier and intra-carrier preference. Hence, the inter-carrier switching will stop.

Therefore, we have prove that the necessary condition of $N$-carrier loop. Together, the conditions (a) and (b) are necessary and sufficient conditions for $N$-carrier loop. □
A.4 Proof of Theorem 6.2

Proof. (Sufficiency ⇒) We will show that under such sufficient condition, \( N \)-carrier loop will happen. We prove it in three main steps.

Step 1. Similar to the proof in Lemma 1 and Theorem 6.1, under any initial condition (\( C_0 \) that phone is connected to), the device will be served by \( C_1 \) in finite steps. Therefore, we will always begin from \( C_1 \).

Step 2. We show that the inter-carrier switching \( C_i \leftrightarrow C_{i+1}, \forall i \in [1, N-1] \) occur. We prove it by induction.

(Base case) First, the switching \( C_1 \leftrightarrow C_2 \) will occur. Similar to the proof of Step 2 for Lemma 1 and Theorem 6.1: first, \( C_1 \) will switch because \( C_1 \)'s intra-carrier logic will lead to an unavailable cell; second, \( C_1 \) will switch to \( C_2 \), but not \( C_3, \ldots, C_N \).

(Inductive step) Next, assume that it is true for \( k, k \in [2, N-2] \) (which means that \( C_k \leftrightarrow C_{k+1} \) occurs), we show that it is true for \( k+1 \). Since \( C_k \leftrightarrow C_{k+1} \) occurs, it means two conditions: (a) \( C_k \) is unavailable, which is assumed by the sufficient condition. (b) \( C_1, C_2, \ldots, C_{k-1} \) have been selected, therefore \( C_{k+1} \) is the highest preference carrier which has not been selected, and has the smallest index among all possible same preference carriers. Together, it proves that \( C_i \leftrightarrow C_{i+1}, \forall i \in [1, N-1] \) occur.

Step 3. We show that the inter-carrier switching \( C_N \leftrightarrow C_1 \) occurs. As \( C_N \) is unavailable assumed by the sufficient condition, it needs to perform inter-carrier switching. Following Steps 1 and 2, we have connected from all carriers \( C_i, \forall i \in [1, N] \). Therefore, all carriers are marked ‘unselected’ again following Policy 2. Therefore, it will select \( C_1 \), since \( P_1 \) is the highest preference and \( C_1 \) is not selected and has the smallest index.

Together, with Steps 1, 2 and 3, we prove that the sufficient condition will lead to an inter-carrier switching sequence \((*) C_1 \leftrightarrow C_2 \leftrightarrow \cdots \leftrightarrow C_N \leftrightarrow C_1 \). With the Lemma 1, the \( N \)-carrier loop occurs.

(Necessity ⇐) We prove by contrapositive. The original statement is: if \( N \)-carrier loop happens, then the necessary condition holds. Therefore we prove the contrapositive statement: if such necessary condition does not hold, \( N \)-carrier loop will not happen.

If the necessary condition does not hold, it means that at least one carrier \( C_i, \exists i \in [1, N] \) will not move the device to an unavailable cell, so that the device has service in carrier \( C_i \). Without loss of generality, \( i \) is the first carrier that will not move the device to an unavailable cell.

Step 1. We first show the inter-carrier switching sequence \((*) C_1 \leftrightarrow C_2 \leftrightarrow \cdots \leftrightarrow C_N \leftrightarrow C_1 \) will not occur under this condition. Following the similar proof to the Lemma 1, it holds. The reason is that inter-carrier switching sequence \( C_1 \leftrightarrow \cdots \leftrightarrow C_i, \exists i \in [1, N] \) will happen, but inter-carrier switching will stop at carrier \( C_i \). The assumption states that \( C_i \) is available while all carriers \( C_1, \ldots, C_{i-1} \) whose preference higher or equal to \( C_i \)'s are unavailable. Following Policy 2, all carriers \( C_1, \ldots, C_{i-1} \) would have been selected when
the serving carrier is $C_i$. Therefore, the highest preference among unselected carriers will be $P_{i+1} \leq P_i$.

Since $C_i$ is available, Policy 2 will decide that staying in $C_i$ ($i \leq N$), so that the inter-carrier switching sequence $(\ast) C_1 \mapsto C_2 \mapsto \cdots \mapsto C_N \mapsto C_1$ does not happen.

Step 2. Following the Lemma 1, since the inter-carrier switching sequence $(\ast) C_1 \mapsto C_2 \mapsto \cdots \mapsto C_N \mapsto C_1$ does not occur, no $N$-carrier loop will happen.

\[ \Box \]

A.5 Proof of Corollary 2

\textbf{Proof.} Due to the similarity of the proof to that of Theorem 6.1, we show a proof sketch here.

**(Sufficiency $\Rightarrow$)** Construct the sequence $(\ast)$ using both conditions. Without loss of generality, suppose $RAT_1$ is $RAT_H$. Further assume that the $RAT_2$ ($RAT_L$ in Theorem 6.1) has the highest intra-carrier priority in all carriers. Step 1, $C_1$ is chosen initially or after finite steps, same reasoning as in Theorem 6.1. Step 2, $C_i \mapsto C_{i+1}$, $\forall i \in [1, N - 1]$ occur. Prove it by induction. The key is, intra-carrier policy always select to $C_i$’s $RAT_2$ by Assumption 1 because highest priority $RAT_2$ is guaranteed to be selected, so $C_i \mapsto C_{i+1}$ happens following condition (a) and Policy 1. Step 3, $C_N \mapsto C_1$ occurs because all carriers are marked ‘unselected’ again following Policy 1. By Lemma 1, an $N$-carrier loop happens since the sequence $(\ast)$ occurs.

**(Necessity $\Leftarrow$)** We prove via contrapositive. First, negate condition (a): if some carriers have no $RAT_H$ assigned with highest preference, then no $N$-carrier loop. It holds because such carrier does not have highest preference, and will not be selected by Policy 1. Second, negate condition (b): if at least in one carrier, most preferred RAT is the same for inter-carrier and intra-carrier policy, then no $N$-carrier loop. It is true because the inter-carrier policy will not further move away, hence it stops. Under both negations, a $k$-carrier loop ($1 < k < N$) may happen, but not the $N$-carrier loop.

\[ \Box \]

B PROOFS OF THEOREMS FOR THRESHOLD-BASED POLICY

Without loss of generality, we assume $M(C_1) \geq M(C_2) \geq \cdots \geq M(C_N)$. Given the problem setting and policy, we have the following Lemma regarding the inter-carrier switching loop.

\textbf{Lemma B.1.} If threshold-policy incurs a $k$-carrier loop ($2 \leq k \leq N$), then it must be $C_1 \mapsto C_2 \mapsto \cdots \mapsto C_k \mapsto C_1$.

B.1 Proof of Theorem 7.1

\textbf{Proof.} Assume inter-carrier policy takes Criterion $F1$ with threshold $\theta$, we prove loop will occur. Based on our problem setting, the threshold must be a reasonable value such that there is chance for any carrier’s measure to be greater than the threshold. Therefore, consider all $N$ carriers have measure greater than threshold, $\theta$. Without the loss of generality, assume $M(C_i) \geq M(C_2) \geq \cdots \geq M(C_N) > \theta$. Since $M(C_i) > \theta (\forall i \in [1, N])$ is satisfied for all carriers at the same time, the phone will keep switching among those carriers.

\[ \Box \]
B.2 Proof of Theorem 7.2

Proof. We prove this theory for Criteria F2–F4 respectively.

F2. (Sufficiency ⇒) Here the measure of the carrier cannot always satisfy \( M(C_j) - M^{\min}(C_j) \leq \phi - \theta \), then we prove the inter-carrier policy cannot be loop-free. Consider only top-\( k \) carriers have the measure \( M \) no less than threshold \( \phi \), i.e. \( C_1, C_2, \ldots, C_k \) and \( M(C_1) \geq M(C_2) \geq \cdots \geq M(C_k) \geq \phi \). In addition, each top-\( k \) carrier \( C_j(1 \leq j \leq k) \) has \( M^{\min}(C_j) < \theta \), which is possible because \( M(C_j) - M^{\min}(C_j) \leq \phi - \theta \) is not always guaranteed. Since the intra-carrier policy is based on a different measure \( Q \) independent of \( M \), in any carrier \( C_j(1 \leq j \leq k) \), the phone could be moved to that cell with measure less than \( \theta \). Initially, assume the phone is connected to a carrier \( C_1 \). Then, based on the inter-carrier switching mechanism and intra-carrier handoff, switchings \( C_i \mapsto C_{i+1}, i \in \{1, k\} \) and \( C_k \mapsto C_1 \) would happen sequentially. By now, a switching loop is formed in static case.

(Necessity ⇐) By setting the measure of any carrier equal to the lowest measure among all its cells, we prove loop-freedom is guaranteed. Once the switching \( C_i \mapsto C_j \) occurs, then there must be \( M(C_j) \geq \phi \). Given \( M(C_j) - M^{\min}(C_j) \leq \phi - \theta \), no matter which cell the intra-carrier handoff leads to, the cell’s measure must be no less than \( M^{\min}(C_j) \geq M(C_j) + \phi - \theta \geq \theta \). As a result, as long as a carrier is selected as the switching target and the phone switches to that carrier, then the phone will not trigger any switching. Loop-freedom is achieved here.

F3, F4. Similar to the proof above.

(Sufficiency ⇒) Since \( M(C_j) - M^{\min}(C_j) > \delta \) is possible for any carrier, we assume there are two carriers \( C_1, C_2 \) which satisfy this condition. When the phone stays on \( C_1 \) or \( C_2 \), intra-carrier handoff will move the phone to the cell with the lowest measure less than \( \theta \). In addition, assume \( M(C_1) = M(C_2) \) and other carriers are unavailable. Under this condition, loop will happen between \( C_1 \) and \( C_2 \) when either F3 or F4 is used.

(Necessity ⇐) Consider \( k \)-carrier loop \( C_1 \mapsto C_2 \mapsto \cdots \mapsto C_k \mapsto C_1 \) occurs. According to Lemma B.1, we have \( M(C_1) \geq M(C_2) \geq \cdots \geq M(C_k) \). Then we have \( M(C_1) \geq M(C_2) > M^{\min}(C_1) + \delta \).

B.3 Proof of Theorem 7.3

Proof. We consider F2 and F4 separately.

F2. We prove loop-freedom is guaranteed if all conditions in Theorem 7.3 are violated. We first prove that, if carrier switching \( C_k \mapsto C_j \) occurs, then the phone would not switch out of \( C_j \) in static case. Given \( C_i \mapsto C_j \), we get \( M(C_j) \geq \phi \). After switching to \( C_j \), the phone initially camps on the cell \( c_j^{u_0} \) with the maximum measure among all cells in \( C_j \), so we have \( M(c_j^{u_0}) \geq M(C_j) \geq \phi \). Finally, the phone is stably connected to cell \( c_j^{u_l} \). So there exists a cell handoff path \( c_j^{u_0} \mapsto c_j^{u_1} \mapsto \cdots \mapsto c_j^{u_l} \) indicating a sequence of cells selected by intra-carrier policy, from the initial cell \( c_j^{u_0} \) till the terminate \( c_j^{u_l} \). Note that handoff may not happen, and \( l \) is possibly equal to 0. Given \( M(c_j^{u_0}) \geq \phi \), we prove any cell in the cell path has measure no less than \( \theta \) if conditions regarding F2 in Theorem 7.3 are violated by \( C_j \).
We prove this by induction. The hypothesis is, for all \( k \in (0, 1] \), if any \( c^u_i (0 \leq i < k) \) has \( M(c^u_i) \geq \theta \) then \( M(c^u_k) \geq \theta \). We have the following cases.

(a) \( k = 0 \). We have \( M(c^u_0) \geq \theta \).

(b) \( k = 1 \). If handoff \( c^u_j \rightarrow c^u_i \) takes criteria of \textit{absolute-value comparison} or \textit{indirect comparison}, then we have \( M(c^u_i) > \text{Thresh}_1^{b-1} \) or \( M(c^u_i) > \text{Thresh}_3^{b-1} \). In both cases, \( M(c^u_i) > \theta \).

(c) \( k \geq 2 \). Suppose for any \( 0 \leq i < k \), \( M(c^u_i) > \theta \) holds. If handoff \( c^u_{k-1} \rightarrow c^u_i \) takes criteria of either \textit{absolute-value comparison} or \textit{indirect comparison}, then we have \( M(c^u_i) \) similar to case (b). Otherwise, handoff \( c^u_{k-1} \rightarrow c^u_i \) takes criteria of \textit{direct comparison}. Analyze the following different cases based on which handoff criteria is used by \( c^u_{k-2} \rightarrow c^u_{k-1} \):

(i) It takes either criteria of \textit{absolute-value comparison} or \textit{indirect comparison}, then we have \( M(c^u_k) > \text{Thresh}_1^{k-2} + \text{Thresh}_3^{k-2} + \Delta_{k-1} \). In both cases, \( M(c^u_k) > \theta \).

(ii) It takes either criteria of \textit{direct comparison}. In this case, \( M(c^u_k) > M(c^u_{k-2}) + \Delta_{k-2} + \Delta_{k-1} \). Since intra-policy is assumed loop-free here, we know \( \Delta_{k-2} + \Delta_{k-1} \geq \theta \) based on the results by Li, et al [2]. So we have \( M(c^u_k) \geq \theta \).

By now, we prove that every cell \( c^u_j \) in the sequence has \( M(c^u_j) \geq \theta \). Therefore, we have \( M(c^u_i) \geq \theta \) so the phone will not switch out of carrier \( C_j \). Since any switching will lead the phone to stay on a new carrier without any more switch, loop would not occur.

F4. We prove this by contradiction.

Assume conditions in Theorem 7.3 are violated and there exists a \( k \)-carrier loop. According to Lemma B.1, the loop is \( C_1 \leftrightarrow C_2 \leftrightarrow \cdots \leftrightarrow C_k \leftrightarrow C_1 \). Within carrier \( C_1 \), assume the handoff sequence is \( c^u_1 \rightarrow c^u_1 \rightarrow \cdots \rightarrow c^u_I, I \geq 0 \). \( c^u_1 \) is the initial cell with \( M(c^u_1) \geq M(C_1) \). Moreover, if handoff happens \( (l > 0) \), then the last handoff must be based on the criterion of \textit{direct comparison}. Otherwise, the phone ends up with a cell whose measure is no less than \( \theta \) and it will not switch out.

Next, we do case analysis on the length of handoff path.

(a) \( l = 0 \). In this case, we have \( M(C_2) \leq M(C_1) \leq M(c^u_1) \). Then the phone will not switch out, so this case is impossible.

(b) \( l = 1 \). In this case, we know handoff \( c^u_1 \rightarrow c^u_1 \) is based on \textit{direct comparison}. Then, \( M(c^u_1) + \delta > M(c^u_1) + \Delta^u + \delta \geq M(C_1) \geq M(C_2) \) shows the phone will not switch out either because the criterion is not satisfied.

(c) \( l \geq 2 \). In the handoff sequence, assume \( c^u_1 \) is the first cell after which all handoffs are based on \textit{direct comparison} criterion. Based on previous analysis, we know \( i \leq l - 1 \).

Here, if \( i = l - 1 \) then handoff \( c^u_{l-1} \rightarrow c^u_{l-1} \) is either based on \textit{absolute-value comparison} or \textit{indirect comparison}, so we have \( M(c^u_{l-1}) > \theta \). In this case, we get either \( M(c^u_{l-1}) > M(c^u_{l-1}) + \Delta_{l-1} \geq \theta \) or \( M(c^u_{l-1}) > M(c^d_{l-1}) + \Delta_{l-1} \geq \text{Thresh}_1^{u_{l-2}} + \Delta_{l-1} \geq \theta \). Both indicate the phone will not switch out of carrier \( C_1 \).
So we only have one case left, that is \( i \leq j \). In this case, \( M(c_1^{u_1}) > M(c_1^{v_1}) + \sum_{x=1}^{l-1} \Delta_1^{w_x} \). Since intra-carrier handoff is assumed loop-free here, and based on the results by Li, et al [2], we have \( \sum_{x=1}^{l-1} \Delta_1^{w_x} \geq 0 \). Therefore, \( M(c_1^{v_1}) > M(c_1^{u_1}) \). Now we know either \( i = 0 \) or not, \( M(c_1^{v_1}) > M(c_1^{u_1}) \geq \min\{\theta, M(c_1^{u_1})\} \). Again, the phone will not switch out. Contradiction.

\[ \square \]

### B.4 Proof of Theorem 7.4

**Proof.** Assume the switching loop is \( C_1 \leftrightarrow C_2 \leftrightarrow \cdots \leftrightarrow C_k, k \in [2, N] \). We have \( M(C_1) \geq M(C_2) \geq \cdots \geq M(C_k) \). Then, we prove \( C_1 \) satisfies the condition in theorem by contradiction. If \( C_1 \) violates the condition, then we show \( C_1 \leftrightarrow C_2 \) would not happen after \( C_k \leftrightarrow C_1 \). The phone switches to \( C_1 \), and initially camps on cell \( c_1^{u_1} \). Based on intra-carrier cell selection policy, the initial cell \( c_1^{u_1} \) has the highest measure among all cells in \( C_1 \). Then intra-carrier handoff may happen and finally move the cell to cell \( c_2^{u_2} \). Next we prove \( M(c_2^{u_2}) \leq M(c_1^{u_1}) + \delta \). (1) If \( c_2^{u_2} \) and \( c_1^{u_1} \) are the same cell, then the condition holds. (2) Otherwise, there is a handoff sequence \( c_1^{u_1} \rightarrow c_2^{u_2} \rightarrow \cdots \rightarrow c_k^{u_k} \). Each handoff in the sequence is based on criterion of direct comparison or indirect comparison. Then we use \( \delta + \sum_{j=0}^{l-1} h(c_j^{u_j} \rightarrow c_j^{u_{j+1}}) \geq 0 \) to prove \( M(c_k^{u_k}) \leq M(c_k^{u_k}) + \delta \). Therefore, we have \( M(C_2) \leq M(C_1) \leq M(c_1^{u_1}) \leq M(c_1^{u_1}) + \delta \). That means the switching \( C_1 \leftrightarrow C_2 \) will not happen because Criterion F3 is not fulfilled. Now we get contradiction.  

\[ \square \]

### C PROOFS OF THEOREMS FOR HYBRID POLICY

#### C.1 Proof of Theorem 8.1

**Proof.** Base on Theorem 7.1, if Criterion F1 is used for the switching \( C_i \leftrightarrow C_j \) and \( C_j \leftrightarrow C_i \) at the same time, then loop will occur. As a result, if F1 is applied to switch between carriers with equal preference, there will be loop. Similarly, if F1 is applied to both switching to higher preference or switching to lower preference, loop will happen too. So far, we have proven combination (1) and (2) are loop-prone.

Next, suppose the switching to a higher preference carrier takes the Criterion F1 and the switching to a lower preference carrier takes Criterion F3. Then we show loop could occur regardless of configuration of threshold. Consider two carriers \( C_1 \) and \( C_2 \) with \( P_1 > P_2 \) and other carriers are unavailable at the current location. Initially the phone stays on \( C_2 \). When \( M(C_1) \geq \theta \), carrier switching \( C_2 \leftrightarrow C_1 \) occurs. In carrier \( C_1 \), the phone is stably connected to cell \( c_1^{x_1} \), while cell \( c_1^{y_1} \) has the maximum measure among all local cells. When \( M(C_2) > M(c_2^{x_2}) + \delta \), carrier switching \( C_1 \leftrightarrow C_2 \) also occurs because \( M(C_2) > M(c_2^{x_2}) + \delta \geq M(c_1^{y_1}) + \delta \). In static case, the phone will keep switching back and forth between \( C_1, C_2 \), which forms loop.

Similarly, we can prove it is also loop-prone to apply F1 to switch to lower preference and F3 to switch to higher preference.  

\[ \square \]
D DYNAMIC POLICY UPDATES

D.1 Proof of Proposition 2

Proof. We prove for each type of update.

(1) Preference update. We prove for RAT-aware preference update here. RAT-oblivious preference update is a special case for this proof.

Suppose the policy update is safe, then the inter-carrier preference values $P_{old}$ given a fixed intra-carrier policy before the update is loop-free by definition. According to Theorem 6.1, $P_{old}$ and the given intra-carrier policy must not satisfy both conditions at the same time: (a) every carrier has one or more RATs (denoted $RAT_H$) assigned with equal and highest preference; and (b) each carrier’s intra-carrier priority and threshold result in reselection from $RAT_H$ to a different $RAT_L$.

Since updating the inter-carrier preference to $P_{new}$ will not affect the given intra-carrier policy, condition (b) is not affected in any case. When the top-preferred $RAT_H$ is given a higher preference, condition (a) will not be satisfied in any case, by enumeration. Therefore, after the update, two conditions still do not satisfy at the same time, thus the loop will not incur by Theorem 6.1. It means that the loop-freedom is still ensured after the policy update under Assumption 1.

(2) Threshold update. We prove the update rule is safe for Criterion $F_2$ respectively. Proof for other criteria is similar.

$F_2$. Suppose the policy is loop-free before update. To update thresholds, we can only decrease $\theta$ or increase $\phi$ or do both. Denote $\theta', \phi'$ as new values, so we have $\theta' \leq \theta, \phi' \geq \phi$. Next we prove that carrier switching which does not happen before update will not happen afterwards either. Consider the phone does not switch from $C_i$ to $C_j$ before. Denote $c_u^i$ as the cell selected as the final serving cell by intra-policy. So the switching criterion is not satisfied, either the $M(c_u^i) \geq \theta$ or $M(C_j) < \phi$. Then, after threshold update, we still have $M(c_u^i) \geq \theta \geq \theta'$ or $M(C_j) < \phi \leq \phi'$. Therefore, switching $C_i \leftrightarrow C_j$ still cannot happen. Then we know, if there exists no loop before threshold update, loop will not happen afterwards as long as the update rule is followed. □

REFERENCES
