# Supplementary Material to Resolving Policy Conflicts in Multi-Carrier Cellular Access

Proofs to the theoretical results in paper: Resolving Policy Conflicts in Multi-Carrier Cellular Access

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In this document, we provide proofs for the theoretical results in the paper [1].

We reiterate the notations (Table 2 in the full paper) for the ease of referring them in the proofs here.

Table 1. Notations

| | |
|---|---|
| $C_i$ | Carrier $i$, $i \in [1, N]$ |
| $RAT_j$ | Radio access technology $j$ (e.g. 3G, 4G) |
| $c^k/c_i^k$ | Cell $k$ (in carrier $C_i$) |
| $P_{i,j}/P_i$ | Inter-carrier preference on carrier $C_i$'s $RAT_j$ / $C_i$ |
| $p(c^i)$ | Intra-carrier priority of cell $c^i$ |
| $M, M(C_i)$ | Measure $M$ (on $C_i$) for inter-carrier policy |
| $Q, q(c^j)$ | Measure $Q$ (on $c^j$) for intra-carrier policy |
| $\delta, \theta, \phi$ | Different inter-carrier thresholds (on carrier) |
| $\Delta^i, Thresh^{i,j}$ | Different intra-carrier thresholds (on $c^i/c^j$) |

## A PROOFS OF THEOREMS FOR PREFERENCE-BASED POLICY

### A.1 Proof of Proposition 1

Proof. Following the static condition assumption, neither inter-carrier policy nor intra-carrier policy changes. Therefore the decision will be the same under deterministic policy, and loop is persistent by definition. □

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### A.2 Proof of Lemma 1

Proof. **(Sufficiency $\Rightarrow$)** The sequence $(*)$ $C_1 \mapsto C_2 \mapsto \cdots \mapsto C_N \mapsto C_1$ by definition is an $N$-carrier loop.

**(Necessity $\Leftarrow$)** We show in three steps that, if an inter-carrier switching sequence contains an $N$-carrier loop, then it contains the sequence $(*)$ $C_1 \mapsto C_2 \mapsto \cdots \mapsto C_N \mapsto C_1$. Suppose the phone initially connects to carrier $C_0$'s $RAT_0$. We denote the highest preference (carrier, RAT) combination in carrier $C_i$ as $P^i_{max} = \max_j P_{i,j}$.

Step 1. We show that under any initial condition ($C_0$ that phone is connected to), the device will be served by $C_1$ after finite switching steps. We prove it by cases. If $C_0 = C_1$, then the conclusion holds. If $C_0 \neq C_1$, then an inter-carrier switching $C_0 \mapsto C_1$ occurs. This is because $P^1_{max} \geq P^0_{max} \geq P_{0,0}$ and $C_1$ is the carrier with the smallest index. According to Policy 1, such switch will happen. Therefore, the device will always be served by $C_1$ initially or after finite steps.

Step 2. The inter-carrier switching from $C_1 \mapsto C_2$ must occur, given that an $N$-carrier loop exists. We can prove it by contradiction: If $C_1 \mapsto C_2$ does not happen, there are two possibilities: either (a) the inter-carrier logic decides to not switch at $C_1$, or that (b) $C_1 \mapsto C_i, i \neq 2$ occurs. For case (a), the conditions are that (1) $C_1$ has the highest preference and (2) $C_1$ is available. Such case does not hold, because though the first condition ($C_1$ has the highest preference) holds by assumption, the second condition does not hold. If it is true, then there is no other possibilities nor reason to switch out from $C_1$, therefore the $N$-carrier loop will not exist. For case (b), this does not hold because $P^1_{max} \geq P^2_{max} \geq P^j_{max}, j \in [3, N]$. Therefore if a switching out from $C_1$ happens, it will switch to $C_2$ according to Policy 1, not any other carrier $C_i, i \neq 2$. Therefore, it must be the case that the inter-carrier switching $C_1 \mapsto C_2$ happens.

Step 3. Similar to the above proof, subsequent inter-carrier switchings $C_2 \mapsto C_3, C_3 \mapsto C_4, \ldots, C_N \mapsto C_1$ must occur in order and no other switching sequences may occur, otherwise the inter-carrier switching sequence will stop at any of the carriers $C_3, C_4, \ldots, C_N$ but no $N$-carrier loop exists. □

### A.3 Proof of Theorem 6.1

Proof. **(Sufficiency $\Rightarrow$)** Suppose both conditions satisfy. Without loss of generality, suppose $RAT_1$ is $RAT_H$ and $RAT_2$ is $RAT_L$. This further implies: (i) for the condition (a) stated in Theorem 6.1, $P_{max} = P^i_{max} = P_{i,1} \geq P_{i,j}, \forall i \in [1, N], \forall j \in [3, N]$, and that $RAT_2$'s preference is always lower than $RAT_1$: $P_{i,2} < P_{i,1}, \forall i \in [1, N]$. (ii) for the condition (b), the intra-carrier logic in every carrier will prefer $RAT_2$: as long as the phone is not connected to $RAT_2$, intra-carrier logic will move phone to $RAT_2$. We next constructively prove that the inter-carrier switching sequence $(*)$ occurs.

Step 1. Starting from $C_0$ and $RAT_0$ initially, we show that phone will be connected to $C_1$ initially or in finite steps. If $C_0 = C_1$ then it is true already; otherwise, suppose $C_0 \neq C_1$ and there are two subcases: (case 1) if $RAT_0 \neq RAT_1$: according to Policy 1, since $P_{0,0} \leq P^0_{max} = P^1_{max} = P_{1,1}$, the inter-carrier switching will select $C_1.RAT_1$ and the phone will connect to $C_1$; (case 2) if $RAT_0 = RAT_1$: according to intra-carrier policy, phone will be reselected to $RAT_2$. Next, inter-carrier policy will select $C_1.RAT_1$ and switch to $C_1$, due to the same reason as (case 1).

Step 2. We show that the inter-carrier switchings $C_i \mapsto C_{i+1}, \forall i \in [1, N-1]$ occur. We prove it by induction.

(Base case) First, the switching $C_1 \mapsto C_2$ will occur. After Step 1, phone is connected to $C_1$. Following condition (b), $C_1$'s intra-carrier policy moves the phone from $RAT_1$ to $RAT_2$. However, since $P_{max} = P_{2,1} > P_{1,2}$ and that $C_2$ is unselected carrier, phone will switch to $C_2$ according to Policy 1. Moreover, $C_1$ will not switch to $C_3, C_4, \ldots, C_N$, because that all these carriers have larger index than $C_2$.

(Inductive step) Next, suppose that it is true for $k, k \in [2, N-2]$ (which means that $C_k \mapsto C_{k+1}$ occurs), we show that it is true for $k+1$. Since $C_k \mapsto C_{k+1}$ occurs, it means two things: (a) Inter-carrier logic chooses $C_k.RAT_1$ according to Policy 1; (b) $C_1, C_2, \ldots, C_k$ have been selected, while $C_{k+1}, C_{k+2}, \ldots$ have not been selected. Given condition (b), intra-carrier logic at $C_{k+1}$ moves to $RAT_2$. Since $P_{max} = P_{k+2,1} > P_{k+1,2}$, inter-carrier logic will perform switch. The switch target is $C_{k+2}$, because $C_1, \ldots, C_{k+1}$ have been selected so that $C_{k+2}$ is the highest preference carrier which has not been selected, and has the smallest index among all possible carriers. Together, $C_i \mapsto C_{i+1}, \forall i \in [1, N-1]$ occur.

Step 3. We show that the inter-carrier switching $C_N \mapsto C_1$ occurs. Following Steps 1 and 2, we have selected all carriers with highest preference: $C_i, \forall i \in [1, N]$. Therefore, when the phone connects to $C_N$, all carriers are marked “unselected” again following Policy 1. When $C_N$'s intra-carrier logic moves phone from $C_N.RAT_1$ to $C_N.RAT_2$, an inter-carrier switching happens because $P_{max} = P_{1,1} = P_{N,1} > P_{N,2}$. Therefore, it will select $C_1.RAT_1$, since it has the highest preference and $C_1$ is not selected and has the smallest index.

Together, with Steps 1, 2 and 3, we prove that the sufficient condition will lead to the inter-carrier switching sequence $(*)$ $C_1 \mapsto C_2 \mapsto \cdots \mapsto C_N \mapsto C_1$. With the Lemma 1, the $N$-carrier loop occurs.

**(Necessity $\Leftarrow$)** We prove via contrapositive. The original statement is: if $N$-carrier loop happens, then both two conditions holds. We prove the contrapositive statement: if one of the conditions does not hold, $N$-carrier loop will not happen.

First, assume the condition (a) does not hold. We are proving: if some carriers have no RAT assigned with highest preference $P_{max}$, then no $N$-carrier loop may happen. It is easy to prove, because the carrier with no RAT assigned with highest preference will not get selected by inter-carrier Policy 1. Under this case, a $k$-carrier loop $(1 < k < N)$ may happen, but not the $N$-carrier loop.

Second, assume the condition (b) does not hold. We are proving: if in some carriers, phone can stay in the $RAT_H$ due to intra-carrier policy, then no $N$-carrier loop may happen. This is evident. When the phone stays in $RAT_H$, it satisfies both inter-carrier and intra-carrier preference. Hence, the inter-carrier switching will stop.

Therefore, we have prove that the necessary condition of $N$-carrier loop. Together, the conditions (a) and (b) are necessary and sufficient conditions for $N$-carrier loop. $\square$

### A.4 Proof of Theorem 6.2

Proof. **(Sufficiency $\Rightarrow$)** We will show that under such sufficient condition, $N$-carrier loop will happen. We prove it in three main steps.

Step 1. Similar to the proof in Lemma 1 and Theorem 6.1, under any initial condition ($C_0$ that phone is connected to), the device will be served by $C_1$ in finite steps. Therefore, we will always begin from $C_1$.

Step 2. We show that the inter-carrier switches $C_i \mapsto C_{i+1}, \forall i \in [1, N-1]$ occur. We prove it by induction.

(Base case) First, the switching $C_1 \mapsto C_2$ will occur. Similar to the proof of Step 2 for Lemma 1 and Theorem 6.1: first, $C_1$ will switch because $C_1$'s intra-carrier logic will lead to an unavailable cell; second, $C_1$ will switch to $C_2$, but not $C_3, \ldots, C_N$.

(Inductive step) Next, assume that it is true for $k, k \in [2, N-2]$ (which means that $C_k \mapsto C_{k+1}$ occurs), we show that it is true for $k+1$. Since $C_k \mapsto C_{k+1}$ occurs, it means two conditions: (a) $C_k$ is unavailable, which is assumed by the sufficient condition. (b) $C_1, C_2, \ldots, C_{k-1}$ have been selected, therefore $C_{k+1}$ is the highest preference carrier which has not been selected, and has the smallest index among all possible same preference carriers.

Therefore, given that $C_{k+1}$ is also unavailable by intra-carrier logic, inter-carrier will perform switching. The switching target is $C_{k+2}$, because $C_1, C_2, \ldots, C_{k-1}, C_k$ have been selected so that $C_{k+2}$ is the highest preference carrier which has not been selected, and has the smallest index among all possible same preference carriers. Together, it proves that $C_i \mapsto C_{i+1}, \forall i \in [1, N-1]$ occur.

Step 3. We show that the inter-carrier switching $C_N \mapsto C_1$ occurs. As $C_N$ is unavailable assumed by the sufficient condition, it needs to perform inter-carrier switching. Following Steps 1 and 2, we have connected from all carriers $C_i, \forall i \in [1, N]$. Therefore, all carriers are marked 'unselected' again following Policy 2. Therefore, it will select $C_1$, since $P_1$ is the highest preference and $C_1$ is not selected and has the smallest index.

Together, with Steps 1, 2 and 3, we prove that the sufficient condition will lead to an inter-carrier switching sequence $(*)$ $C_1 \mapsto C_2 \mapsto \cdots \mapsto C_N \mapsto C_1$. With the Lemma 1, the $N$-carrier loop occurs.

**(Necessity $\Leftarrow$)** We prove by contrapositive. The original statement is: if $N$-carrier loop happens, then the necessary condition holds. Therefore we prove the contrapositive statement: if such necessary condition does not hold, $N$-carrier loop will not happen.

If the necessary condition does not hold, it means that at least one carrier $C_i, \exists i \in [1, N]$ will not move the device to an unavailable cell, so that the device has service in carrier $C_i$. Without loss of generality, $i$ is the *first* carrier that will not move the device to an unavailable cell.

Step 1. We first show the inter-carrier switching sequence $(*)$ $C_1 \mapsto C_2 \mapsto \cdots \mapsto C_N \mapsto C_1$ will not occur under this condition. Following the similar proof to the Lemma 1, it holds. The reason is that inter-carrier switching sequence $C_1 \mapsto \cdots \mapsto C_i, \exists i \in [1, N]$ will happen, but inter-carrier switching will stop at carrier $C_i$. The assumption states that $C_i$ is available while all carriers $C_1, \ldots, C_{i-1}$ whose preference higher or equal to $C_i$'s are unavailable. Following Policy 2, all carriers $C_1, \ldots, C_{i-1}$ would have been selected when the serving carrier is $C_i$. Therefore, the highest preference among unselected carriers will be $P_{i+1} \leq P_i$. Since $C_i$ is available, Policy 2 will decide that staying in $C_i$ ($i \leq N$), so that the inter-carrier switching sequence $(*)\ C_1 \mapsto C_2 \mapsto \cdots \mapsto C_N \mapsto C_1$ does not happen.

Step 2. Following the Lemma 1, since the inter-carrier switching sequence $(*)\ C_1 \mapsto C_2 \mapsto \cdots \mapsto C_N \mapsto C_1$ does not occur, no $N$-carrier loop will happen. □

### A.5 Proof of Corollary 2

Proof. Due to the similarity of the proof to that of Theorem 6.1, we show a proof sketch here.

**(Sufficiency ⇒)** Construct the sequence $(*)$ using both conditions. Without loss of generality, suppose $RAT_1$ is $RAT_H$. Further assume that the $RAT_2$ ($RAT_L$ in Theorem 6.1) has the highest intra-carrier priority in all carriers. Step 1, $C_1$ is chosen initially or after finite steps, same reasoning as in Theorem 6.1. Step 2, $C_i \mapsto C_{i+1}, \forall i \in [1, N-1]$ occur. Prove it by induction. The key is, intra-carrier policy always select to $C_i$'s $RAT_2$ by Assumption 1 because highest priority $RAT_2$ is guaranteed to be selected, so $C_i \mapsto C_{i+1}$ happens following condition (a) and Policy 1. Step 3, $C_N \mapsto C_1$ occurs because all carriers are marked 'unselected' again following Policy 1. By Lemma 1, an $N$-carrier loop happens since the sequence $(*)$ occurs.

**(Necessity ⇐)** We prove via contrapositive. First, negate condition (a): if some carriers have no $RAT_H$ assigned with highest preference, then no $N$-carrier loop. It holds because such carrier does not have highest preference, and will not be selected by Policy 1. Second, negate condition (b): if at least in one carrier, most preferred RAT is the same for inter-carrier and intra-carrier policy, then no $N$-carrier loop. It is true because the inter-carrier policy will not further move away, hence it stops. Under both negations, a $k$-carrier loop ($1 < k < N$) may happen, but not the $N$-carrier loop. □

## B PROOFS OF THEOREMS FOR THRESHOLD-BASED POLICY

Without loss of generality, we assume $M(C_1) \geq M(C_2) \geq \cdots \geq M(C_N)$. Given the problem setting and policy, we have the following Lemma regarding the inter-carrier switching loop.

Lemma B.1. *If threshold-policy incurs a $k$-carrier loop ($2 \leq k \leq N$), then it must be $C_1 \mapsto C_2 \mapsto \cdots \mapsto C_k \mapsto C_1$.*

### B.1 Proof of Theorem 7.1

Proof. Assume inter-carrier policy takes Criterion *F1* with threshold $\theta$, we prove loop will occur. Based on our problem setting, the threshold must be a reasonable value such that there is chance for any carrier's measure to be greater than the threshold. Therefore, consider all $N$ carriers have measure greater than threshold, $\theta$. Without the loss of generality, assume $M(C_1) \geq M(C_2) \geq \cdots \geq M(C_N) > \theta$. Since $M(C_i) > \theta (\forall i \in [1, N])$ is satisfied for all carriers at the same time, the phone will keep switching among those carriers. □

### B.2 Proof of Theorem 7.2

Proof. We prove this theory for Criteria *F2–F4* respectively.

**F2. (Sufficiency $\Rightarrow$)** Here the measure of the carrier cannot always satisfy $M(C_j) - M^{min}(C_j) \leq \phi - \theta$, then we prove the inter-carrier policy cannot be loop-free. Consider only top-$k$ carriers have the measure $M$ no less than threshold $\phi$, i.e. $C_1, C_2, \cdots, C_k$ and $M(C_1) \geq M(C_2) \geq \cdots \geq M(C_k) \geq \phi$. In addition, each top-$k$ carrier $C_j (1 \leq j \leq k)$ has $M^{min}(C_j) < \theta$, which is possible because $M(C_j) - M^{min}(C_j) \leq \phi - \theta$ is not always guaranteed. Since the intra-carrier policy is based on a different measure $Q$ independent of $M$, in any carrier $C_j (1 \leq j \leq k)$, the phone could be moved to that cell with measure less than $\theta$. Initially, assume the phone is connected to a carrier $C_1$. Then, based on the inter-carrier switching mechanism and intra-carrier handoff, switchings $C_i \mapsto C_{i+1}, i \in [1, k)$ and $C_k \mapsto C_1$ would happen sequentially. By now, a switching loop is formed in static case.

**(Necessity $\Leftarrow$)** By setting the measure of any carrier equal to the lowest measure among all its cells, we prove loop-freedom is guaranteed. Once the switching $C_i \mapsto C_j$ occurs, then there must be $M(C_j) \geq \phi$. Given $M(C_j) - M^{min}(C_j) \leq \phi - \theta$, no matter which cell the intra-carrier handoff leads to, the cell's measure must be no less than $M^{min}(C_j) \geq M(C_j) + \phi - \theta \geq \theta$. As a result, as long as a carrier is selected as the switching target and the phone switches to that carrier, then the phone will not trigger any switching. Loop-freedom is achieved here.

**F3, F4.** Similar to the proof above.

**(Sufficiency $\Rightarrow$)** Since $M(C_j) - M^{min}(C_j) > \delta$ is possible for any carrier, we assume there are two carriers $C_1, C_2$ which satisfy this condition. When the phone stays on $C_1$ or $C_2$, intra-carrier handoff will move the phone to the cell with the lowest measure less than $\theta$. In addition, assume $M(C_1) = M(C_2)$ and other carriers are unavailable. Under this condition, loop will happen between $C_1$ and $C_2$ when either *F3* or *F4* is used.

**(Necessity $\Leftarrow$)** Consider $k$-carrier loop $C_1 \mapsto C_2 \mapsto \cdots \mapsto C_k \mapsto C_1$ occurs. According to Lemma B.1, we have $M(C_1) \geq M(C_2) \geq \cdots \geq M(C_k)$. Then we have $M(C_1) \geq M(C_2) > M^{min}(C_1) + \delta$. □

### B.3 Proof of Theorem 7.3

Proof. We consider *F2* and *F4* separately.

**F2.** We prove loop-freedom is guaranteed if all conditions in Theorem 7.3 are violated. We first prove that, if carrier switching $C_{j_0} \mapsto C_j$ occurs, then the phone would not switch out of $C_j$ in static case. Given $C_i \mapsto C_j$, we get $M(C_j) \geq \phi$. After switching to $C_j$, the phone initially camps on the cell $c_j^{u_0}$ with the maximum measure among all cells in $C_j$, so we have $M(c_j^{u_0}) \geq M(C_j) \geq \phi$. Finally, the phone is stably connected to cell $c_j^{u_l}$. So there exists a cell handoff path $c_2^{u_0} \rightarrow c_2^{u_1} \rightarrow \cdots \rightarrow c_2^{u_l}$ indicating a sequence of cells selected by intra-carrier policy, from the initial cell $c_j^{u_0}$ till the terminate $c_j^{u_l}$. Note that handoff may not happen, and $l$ is possibly equal to 0. Given $M(c_j^{u_0}) \geq \phi$, we prove any cell in the cell path has measure no less than $\theta$ if conditions regarding *F2* in Theorem 7.3 are violated by $C_j$.

We prove this by induction. The hypothesis is, for all $\forall k \in (0, l]$, if any $c_j^{u_i} (0 \le i < k)$ has $M(c_j^{u_i}) \ge \theta$ then $M(c_j^{u_k}) \ge \theta$. We have the following cases.

(a) $k = 0$. We have $M(c_j^{u_0}) \ge \phi \ge \theta$.

(b) $k = 1$. If handoff $c_j^{u_0} \longrightarrow c_j^{u_1}$ takes criteria of *absolute-value comparison* or *indirect comparison*, then we have $M(c_j^{u_1} > Thresh1_j^{0,1})$ or $M(c_j^{u_1} > Thresh3_j^{0,1})$. In both cases, $M(c_j^{u_1}) > \theta$.

(c) $k \ge 2$. Suppose for any $0 \le i < k$, $M(c_j^{u_i}) > \theta$ holds. If handoff $c_j^{u_{k-1}} \longrightarrow c_j^{u_k}$ takes criteria of either *absolute-value comparison* or *indirect comparison*, then we have $M(c_j^{u_k})$ similar to case (b). Otherwise, handoff $c_j^{u_{k-1}} \longrightarrow c_j^{u_k}$ takes criteria of *direct comparison*. Analyze the following different cases based on which handoff criteria is used by $c_j^{u_{k-2}} \longrightarrow c_j^{u_{k-1}}$:
   - (i) It takes either criteria of *absolute-value comparison* or *indirect comparison*, then we have $M(c_j^{u_k} > Thresh1_j^{k-2,k-1} + \Delta_j^{k-1})$ or $M(c_j^{u_k} > Thresh3_j^{k-2,k-1} + \Delta_j^{k-1})$. In both cases, $M(c_j^{u_k}) > \theta$.
   - (ii) It takes either criteria of *direct comparison*. In this case, $M(c_j^{u_k}) > M(c_j^{u_{k-2}}) + \Delta_j^{k-2} + \Delta_j^{k-1}$. Since intra-policy is assumed loop-free here, we know $\Delta_j^{k-2} + \Delta_j^{k-1} \ge 0$ based on the results by Li, et al [2]. So we have $M(c_j^{u_k}) > M(c_j^{u_{k-2}}) \ge \theta$.

By now, we prove that every cell $c_j^{u_i}$) in the sequence has $M(c_j^{u_i}) \ge \theta$. Therefore, we have $M(c_j^{u_l}) \ge \theta$ so the phone will not switch out of carrier $C_j$. Since any switching will lead the phone to stay on a new carrier without any more switch, loop would not occur.

**F4.** We prove this by contradiction.

Assume conditions in Theorem 7.3 are violated and there exists a $k$-carrier loop. According to Lemma B.1, the loop is $C_1 \mapsto C_2 \mapsto \cdots \mapsto C_k \mapsto C_1$. Within carrier $C_1$, assume the handoff sequence is $c_1^{u_0} \longrightarrow c_1^{u_1} \longrightarrow \cdots \longrightarrow c_1^{u_l}, l \ge 0$. $c_1^{u_0}$ is the initial cell with $M(c_1^{u_0}) \ge M(C_1)$. Moreover, if handoff happens ($l > 0$), then the last handoff must be based on the criterion of *direct comparison*. Otherwise, the phone ends up with a cell whose measure is no less than $\theta$ and it will not switch out.

Next, we do case analysis on the length of handoff path.

(a) $l = 0$. In this case, we have $M(C_2) \le M(C_1) \le M(c_1^{u_0})$. Then the phone will not switch out, so this case is impossible.

(b) $l = 1$. In this case, we know handoff $c_1^{u_0} \longrightarrow c_1^{u_1}$ is based on *direct* comparison. Then, $M(c_1^{u_1}) + \delta > M(c_1^{u_0}) + \Delta^{u_0} + \delta \ge M(c_1^{u_0}) \ge M(C_1) \ge M(C_2)$ shows the phone will not switch out either because the criterion is not satisfied.

(c) $l \ge 2$. In the handoff sequence, assume $c_1^{u_i}$ is the first cell after which all handoffs are based on *direct comparison* criterion. Based on previous analysis, we know $i \le l - 1$.
Here, if $i = l - 1$ then handoff $c_1^{u_{l-2}} \longrightarrow c_1^{u_{l-1}}$ is either based on *absolute-value comparison* or *indirect comparison*, so we have $M(c_1^{u_i}) > \theta$. In this case, we get either $M(c_1^{u_l}) > M(c_1^{u_{l-1}}) + \Delta_1^{u_{l-1}} > Thresh1_1^{u_{l-2},u_{l-1}} + \Delta_1^{u_{l-1}} \ge \theta$ or $M(c_1^{u_l}) > M(c_1^{u_{l-1}}) + \Delta_1^{u_{l-1}} > Thresh3_1^{u_{l-2},u_{l-1}} + \Delta_1^{u_{l-1}} \ge \theta$. Both indicate the phone will not switch out of carrier $C_1$.

So we only have one case left, that is $i \leq l-2$. In this case, $M(c_1^{u_l}) > M(c_1^{u_i}) + \sum_{x=i}^{l-1} \Delta_1^{u_x}$. Since intra-carrier handoff is assumed loop-free here, and based on the results by Li, et al [2], we have $\sum_{x=i}^{l-1} \Delta_1^{u_x} \geq 0$. Therefore, $M(c_1)^{u_l} > M(c_1)^{u_i}$. Now we know either $i = 0$ or not, $M(c_1)^{u_l} > M(c_1)^{u_i} \geq \min\{\theta, M(c_1^{u_0})\}$. Again, the phone will not switch out. Contradiction.

□

### B.4 Proof of Theorem 7.4

Proof. Assume the switching loop is $C_1 \mapsto C_2 \mapsto \cdots C_k, k \in [2, N]$. We have $M(C_1) \geq M(C_2) \geq \cdots \geq M(C_k)$. Then, we prove $C_1$ satisfies the condition in theorem by contradiction. If $C_1$ violates the condition, then we show $C_1 \mapsto C_2$ would not happen after $C_k \mapsto C_1$. The phone switches to $C_1$, and initially camps on cell $c_1^{u_0}$. Based on intra-carrier cell selection policy, the initial cell $c_1^{u_0}$ has the highest measure among all cells in $C_1$. Then intra-carrier handoff may happen and finally move the cell to cell $c_1^{u_l}$. Next we prove $M(c_1^{u_0}) \leq M(c_1^{u_l}) + \delta$. (1) If $c_1^u$ and $c_1^{u_0}$ are the same cell, then the condition holds. (2) Otherwise, there is a handoff sequence $c_1^{u_0} \to c_1^{u_1} \to \cdots c_1^{u_l}$. Each handoff in the sequence is based on criterion of direct comparison or indirect comparison. Then we use $\delta + \sum_{j=0}^{l-1} h(c_1^{u_j} \to c_1^{u_{j+1}}) \geq 0$ to prove $M(c_1^{u_0}) \leq M(c_1^{u_l}) + \delta$. Therefore, we have $M(C_2) \leq M(C_1) \leq M(c_1^{u_0}) \leq M(c_1^{u_l}) + \delta$. That means the switching $C_1 \mapsto C_2$ will not happen because Criterion *F3* is not fulfilled. Now we get contradiction. □

## C PROOFS OF THEOREMS FOR HYBRID POLICY

### C.1 Proof of Theorem 8.1

Proof. Base on Theorem 7.1, if Criterion *F1* is used for the switching $C_i \mapsto C_j$ and $C_j \mapsto C_i$ at the same time, then loop will occur. As a result, if *F1* is applied to switch between carriers with equal preference, there will be loop. Similarly, if *F1* is applied to both switching to higher preference or switching to lower preference, loop will happen too. So far, we have proven combination (1) and (2) are loop-prone.

Next, suppose the switching to a higher preference carrier takes the Criterion *F1* and the switching to a lower preference carrier takes Criterion *F3*. Then we show loop could occur regardless of configuration of threshold. Consider two carriers $C_1$ and $C_2$ with $P_1 > P_2$ and other carriers are unavailable at the current location. Initially the phone stays on $C_2$. When $M(C_1) \geq \theta$, carrier switching $C_2 \mapsto C_1$ occurs. In carrier $C_1$, the phone is stably connected to cell $c_1^u$, while cell $c_1^v$ has the maximum measure among all local cells. When $M(C_2) > M(c_1^v) + \delta$, carrier switching $C_1 \mapsto C_2$ also occurs because $M(C_2) > M(c_1^v) + \delta \geq M(c_1^u) + \delta$. In static case, the phone will keep switching back and forth between $C_1, C_2$, which forms loop.

Similarly, we can prove it is also loop-prone to apply *F1* to switch to lower preference and *F3* to switch to higher preference. □

# D DYNAMIC POLICY UPDATES

## D.1 Proof of Proposition 2

Proof. We prove for each type of update.

**(1) Preference update.** We prove for RAT-aware preference update here. RAT-oblivious preference update is a special case for this proof.

Suppose the policy update is *safe*, then the inter-carrier preference values $P_{old}$ given a fixed intra-carrier policy *before* the update is loop-free by definition. According to Theorem 6.1, $P_{old}$ and the given intra-carrier policy *must not* satisfy both conditions at the same time: (a) every carrier has one or more RATs (denoted $RAT_H$) assigned with equal and highest preference; and (b) each carrier's intra-carrier priority and threshold result in reselection from $RAT_H$ to a different $RAT_L$.

Since updating the inter-carrier preference to $P_{new}$ will not affect the given intra-carrier policy, condition (b) is not affected in any case. When the top-preferred $RAT_H$ is given a higher preference, condition (a) will not be satisfied in any case, by enumeration. Therefore, after the update, two conditions still do not satisfy *at the same time*, thus the loop will not incur by Theorem 6.1. It means that the loop-freedom is still ensured *after* the policy update under Assumption 1.

**(2) Threshold update.** We prove the update rule is *safe* for Criterion *F2* respectively. Proof for other criteria is similar.

**F2.** Suppose the policy is loop-free before update. To update thresholds, we can only decrease $\theta$ or increase $\phi$ or do both. Denote $\theta', \phi'$ as new values, so we have $\theta' \leq \theta, \phi' \geq \phi$. Next we prove that carrier switching which does not happen before update will not happen afterwards either. Consider the phone does not switch from $C_i$ to $C_j$ before. Denote $c_i^u$ as the cell selected as the final serving cell by intra-policy. So the switching criterion is not satisfied, either the $M(c_i^u) \geq \theta$ or $M(C_j) < \phi$. Then, after threshold update, we still have $M(c_i^u) \geq \theta \geq \theta'$ or $M(C_j) < \phi \leq \phi'$. Therefore, switching $C_i \mapsto C_j$ still cannot happen. Then we know, if there exists no loop before threshold update, loop will not happen afterwards as long as the update rule is followed. □

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