Downlink MIMO with Frequency-Domain Packet Scheduling for 3GPP LTE

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***Abstract*—This paper addresses the problem of frequency domain packet scheduling (FDPS) incorporating spatial division multiplexing (SDM) multiple input multiple output (MIMO) techniques on the 3GPP Long Term Evolution (LTE) downlink. We impose the LTE MIMO constraint of selecting only one MIMO mode (spatial multiplexing or transmit diversity) per user per transmission time interval (TTI). First, we address the optimal MIMO mode selection (multiplexing or diversity) per user in each TTI in order to maximize the proportional fair (PF) criterion extended to frequency and spatial domains. We prove that the SU-MIMO (single-user MIMO) FDPS problem under the LTE requirement is NP-hard and therefore, we develop two approximation algorithms (one with full channel feedback and the other with partial channel feedback) with provable performance bounds. Based on 3GPP LTE system model simulations, the approximation algorithm with partial channel feedback is shown to have comparable performance to the one with full channel feedback, while significantly reducing the channel feedback overhead by nearly 50%.**

I. Introduction

The Third Generation Partnership Project (3GPP) Long-Term Evolution (LTE) standardization efforts aim at developing future cellular technologies in order to improve spectral efficiency and coverage while reducing costs [1]. Orthogonal Frequency Division Multiple Access (OFDMA) has been selected for LTE downlink (DL) radio access scheme due to its robustness to multipath fading, higher spectral efficiency and bandwidth scalability. Multiple access in DL OFDMA is achieved by assigning different frequency portions of the system bandwidth to individual users based on their existing channel conditions. In LTE DL, the system bandwidth is divided into multiple subbands (i.e. groups of subcarriers) denoted as *resource blocks* (RBs). A resource block is the minimum scheduling resolution in the time-frequency domain.

In order to schedule resources to the different users on the downlink, the base station (or eNodeB) needs channel quality reports from the individual users. The feedback reports from the individual terminal users to the base station are sent in the form of a *channel quality indicator* (CQI). A CQI is an estimate of the downlink channel at the individual users and is obtained using reference signals transmitted from the base station. The packet scheduler at the base station uses the CQI feedback from individual users to perform an RB to user assignment every transmission time interval (TTI of 1ms in LTE) according to the base station selected scheduling policy. The scheduler also determines the data rate to be used for each user in each subframe and can perform rate adaptation by using *adaptive modulation and coding* (AMC) in different subframes [5]. Such fast *channel dependent scheduling* in both time and frequency domain multiplexing is referred to as *frequency-domain packet scheduling* (FDPS) [15]. Recent studies [15]–[17], [21] have shown potential gains in system capacity of up to 40-60% over time-domain only scheduling.

Another promising technology for LTE is the use of multiple input multiple output (MIMO) antennas that can further improve the spectral efficiency gain by using *spatial division multiplexing* [3]. Multiple antennas allow for an additional degree of freedom to the channel scheduler. Different MIMO schemes are considered in the 3GPP standard depending on the spatial domain user selection over individual RBs. Single-user MIMO (SU-MIMO) has the restriction that only one user can be scheduled (as either transmit diversity mode or spatial multiplexing mode) over each RB. Multi-user MIMO (MU-MIMO) offers greater spatial-domain flexibility by allowing different users to be scheduled on different spatial streams over the same RB. In this paper, we focus on the SU-MIMO scheduler.

In order to achieve large MIMO FDPS gain by exploiting spatial, frequency and multiuser diversity, the scheduler needs to know the instantaneous radio channel conditions across all users, RBs, and spatial streams for all the available MIMO modes. Hence, with full CQI feedback, each user reports three CQIs per RB; one for single-stream diversity mode and one each for the two individual streams for dual-stream spatial multiplexing MIMO mode [19], [20].

A straightforward approach for MIMO-FDPS is to select the best user and corresponding MIMO mode (tx diversity or multiplexing) for each individual RB independently by considering a single RB in isolation regardless of other RBs' assignment status. However, such a scheduling strategy cannot be employed in the 3GPP LTE system which constrains downlink transmission to each user terminal to only one MIMO mode within each TTI (all RBs in a subframe assigned to an individual user is transmitted either using transmit diversity or by using spatial multiplexing) in order to reduce signaling overhead [3], [4]. Therefore, the LTE DL MIMO FDPS algorithms need to incorporate this MIMO mode constraint while trying to maximize the scheduling objective.

In this paper, we address the fundamental MIMO-FDPS problem of performing an optimal (depending on the scheduling objective) RB to user mapping over the entire set of resource blocks available at the downlink along with an additional constraint of restricting the transmission of all the RBs selected for an individual user in a TTI (1ms) to one MIMO mode. We analyze this problem by adopting the well-known time-domain *Proportional Fair* (PF) algorithm to maximize the proportional fair criteria in the MIMO-FDPS setting. The main goal of this paper is to extend the time-domain PF algorithm to this problem framework by incorporating the additional frequency and spatial dimensions.

A. The Model

We consider a cellular network with a single base station and n active wireless users, each with two transmit and two receive antennas (i.e. 2×2 MIMO antenna scheme). The system bandwidth is divided into m RBs i.e., the base station can allocate m RBs to a set of n users. At each time instance multiple RBs can be assigned to a single user (with the only one MIMO mode constraint), each RB however can be assigned to at most one user (as single-stream diversity mode or dual-stream spatial multiplexing MIMO mode). In this paper we assume an *infinitely backlogged* model, i.e. at each time instance, the base station has data available for transmission to every user. Thus, the base station can schedule all the m RBs at every time instance (i.e. TTI of 1 ms in LTE).

We define the indicator variable $x^c_{i,j}(t)$ to indicate whether or not RB c is assigned to user i with MIMO mode j (D: Diversity, M: Multiplexing) at time instance t. We assume that the channel conditions vary across different RBs and for different users. The channel conditions vary with time, frequency (e.g. *frequency selective multipath fading*) and user location. Therefore, each RB has a corresponding *user-dependent* and *time-varying* channel condition that is represented by the CQI for that user over that RB. Let $r^c_{i,j}(t)$ be the transmit data size for user i with MIMO mode j on RB c at time t, which can be calculated from the CQI feedback. The feedback from a user consists of three elements per RB: rank index (RI), precoding matrix index (PMI), and CQI(s) itself [3]. We assume that the base station estimates the signal to noise ratio (SNR) on each spatial RB for each user based on the user feedback.

Let $p_{e,D}(\mathsf{SNR})$ and $p_{e,M}(\mathsf{SNR})$ be the block error rates (BLER) corresponding to the estimated SNR for a given CQI using transmit diversity and spatial multiplexing, respectively. The SNR-BLER modeling for different MIMO transmission schemes can be obtained by prior link-level simulations and made available at the base station. In an effort to make the optimal use of MIMO channels, we utilize the concept of effective data rate $\hat{r}^c_{i,j}(t)$, which takes into account the transmit data size $r^c_{i,j}(t)$ and the BLER for user i for MIMO mode j on RB c at time t:

$$\hat{r}^c_{i,j}(t) = r^c_{i,j}(t) \times \Big(1 - p_{e,j}(\mathsf{SNR})\Big)$$

Note that for a given user i and RB c, the effective data rate $\hat{r}^c_{i,j}(t)$ shows a *diversity-multiplexing tradeoff* [22] since the diversity mode offers more reliable transmission (i.e. lower BLER) for the same modulation and coding scheme (MCS) while the transmit data size $r^c_{i,j}(t)$ for multiplexing MIMO mode is generally larger than the one for diversity mode (i.e. $r^c_{i,M}(t) \geq r^c_{i,D}(t)$) due to its dual-stream spatial multiplexing transmission. Thus, if $x^c_{i,j}(t) = 1$, then user i has an *effective* data rate of $\hat{r}^c_{i,j}(t)$ for RB c with MIMO mode j at time instance t.

B. Problem Formulation

In the time-domain, the well known Proportional Fair (PF) algorithm [12], [18] aim to maximize over all feasible scheduling rules, the utility function $\sum_i \log R_i$ (known as as *proportional fair criteria*), where R_i is the long-term service rate of user i, and updated according to:

$$R_i(t+1) = \begin{cases} (1-\alpha)R_i(t) + \alpha r_i(t) & \text{if user } i \text{ scheduled} \\ (1-\alpha)R_i(t) & \text{otherwise.} \end{cases}$$

where $r_i(t)$ is the channel rate for user i and α is a time constant typically on the order of 1000 slots (e.g. 1/1000). In order to maximize $\sum_i \log R_i$, one should maximize $\sum_i d_i(t)/R_i(t)$ where $d_i(t)$ is total data transmitted to user i at time t (proven in [6], [14], [18]). Hence the time-domain PF algorithm always serves the user who maximizes $r_i(t)/R_i(t)$ at each time step t. Note that the PF algorithm achieves high throughput and maintains proportional fairness amongst all users by giving priority to users with a high-quality channel rate ($r_i(t)$) and a low current average service rate ($R_i(t)$). In this way the PF scheduler strikes a good balance between overall throughput and fairness.

We now extend the time-domain PF algorithm for the MIMO-FDPS scheduler by taking into account the extra frequency and spatial dimensions. We again aim at maximizing the utility function $\sum_i \log R_i$ where the objective R_i is updated according to:

$$R_i(t+1) = (1-\alpha)R_i(t) + \alpha \sum_c \sum_j x^c_{i,j}(t) \cdot \hat{r}^c_{i,j}(t)$$

Note that R_i is updated with the effective data rate $\hat{r}^c_{i,j}(t)$ instead of the transmit data size $r^c_{i,j}(t)$, since we aim to make the best use of MIMO channels by exploiting the diversity-multiplexing tradeoff, which is represented by the effective data rate $\hat{r}^c_{i,j}(t)$.

Let $\lambda^c_{i,j}(t) = \hat{r}^c_{i,j}(t)/R_i(t)$ be the *PF metric value* for user i on RB c and MIMO mode j at time instance t. Based on the work in [9], we define a MIMO FDPS version of PF objective function at time instance t as follows:

$$\max \sum_i \sum_c \sum_{j \in \{D,M\}} x^c_{i,j}(t) \lambda^c_{i,j}(t) \quad (1)$$

It is fairly straightforward to see that objective (1) maximizes $\sum_i d_i(t)/R_i(t)$ at time step t, and therefore achieves proportional fairness, i.e. optimizing objective (1) maximizes the utility function $\sum_i \log R_i$ in the time, frequency, and spatial domain context. For this reason, a straightforward approach for MIMO FDPS scheduling is to apply the PF algorithm directly over each RB one-by-one, i.e. for RB c the PF algorithm selects the best user and corresponding MIMO mode (tx diversity or multiplexing) maximizing $\hat{r}^c_{i,j}(t)/R_i(t)$ at time slot t. However, for LTE DL MIMO FDPS scheduling we need to incorporate the additional constraint of using only one MIMO mode per user per time instance. Accordingly, we can rewrite the objective (1) as the following optimization problem:

$$\max \sum_i \sum_c \sum_{j \in \{D,M\}} x^c_{i,j} \lambda^c_{i,j} \tag{1}$$

subject to

$$\sum_i \sum\nolimits_{j \in \{D,M\}} x^c_{i,j} \leq 1, \quad \forall c \tag{2}$$

$$\sum_i \sum_c \sum_j x^c_{i,j} \leq m \tag{3}$$

$$i \neq i', \qquad \forall c, c', x^c_{i,D} = x^{c'}_{i',M} = 1 \tag{4}$$

$$x^c_{i,j} \in \{0,1\} \tag{5}$$

To simplify notation, the dependence on time t is omitted. Constraint (2) states that each RB can be assigned to at most one user either using transmit diversity or spatial multiplexing. Constraint (3) denotes that the system has the total of m RBs. Constraint (4) ensures that only one MIMO mode (i.e. diversity or multiplexing) can be selected for a single user across different RBs for that user for each time instance. In this paper, we explore the fundamental nature of this MIMO-FDPS problem by seeking an efficient algorithm that maximizes objective (1) while satisfying constraints (2)-(5).

C. *Related work*

The Proportional Fair (PF) algorithm was introduced in [12], [18] and has been extensively studied in the research community [6]–[8], and is widely used as a standard scheduling algorithm in the current wireless systems such as CDMA 2000 1xEV-DO [10], [12].

The FDPS scheduling research is still in a preliminary stage, and most studies directly adapt the time-domain PF algorithm into frequency-domain context. Results show that the FDPS promises up to 40-60% gain over time-domain only scheduling [15]–[17]. Moreover [21] shows that the frequency selectivity of FDPS significantly improves the short-term fairness (a well-known problem of the conventional time-domain PF scheduling is its poor short-term fairness).

There have been very few research results on extending the FDPS algorithm using the additional spatial degree of freedom by incorporating multiple antennas at the transmitter and receiver. In [19], the authors raise the issue of the MIMO mode constraint in LTE and proposed a couple of simple heuristic algorithms. However, they provide neither the analysis on the hardness implication of this problem, nor theoretical performance bounds of the algorithms. In [20], signaling reduction methods are proposed to mitigate the CQI reporting overhead in the context of MIMO-FDPS.

In this paper, we extend the LTE FDPS scheduling algorithm to incorporate the MIMO mode selection (diversity or multiplexing) per user in each TTI in order to maximize the PF criterion extended to frequency and spatial domains. We prove that the SU-MIMO FDPS problem with the only one MIMO mode constraint is NP-hard. We develop two approximation algorithms ($Alg1$ with full channel feedback and $Alg2$ with partial channel feedback) with provable performance bounds. Using 3GPP LTE system model simulations, we show that both algorithms offer measurable throughput gains over 1×2 SIMO FDPS-only scheduling. However, the short-term fairness of the FDPS-only scheduling is better compared to the SU-MIMO FDPS scheduling algorithms. Finally, we show that although $Alg2$ has partial feedback information (nearly 50% feedback reduction compared to $Alg1$), its performance is comparable to $Alg1$.

II. Hardness Result

In this section, we first show that the SU-MIMO FDPS problem is NP-hard and hence, we cannot find an efficient algorithm that optimizes objective (1) under the only one MIMO mode constraint unless P = NP. We then demonstrate that the search space for the above problem is quite large making it computationally intractable for practical systems.

A. *Hardness of objective (1)*

For our reduction, we present the NP-complete problem called *3-Satisfiability*, or *3-SAT* [13]:

- Given a set of clauses $C_1, \ldots, C_k$, each of length 3, over a set of boolean variables $X = \{x_1, \ldots, x_n\}$, does there exist a satisfying truth assignment?

In 3-SAT, a *clause* is simply a disjunction of 3 distinct terms (i.e. $t_1 \vee t_2 \vee t_3$, each term $t_i \in \{x_1, x_2, \ldots, x_n, \overline{x_1}, \ldots, \overline{x_n}\}$), and a *truth assignment* for X is an assignment of the value 0 or 1 to each x_i. An assignment *satisfies* a collection of clauses $C_1, \ldots, C_k$ if it causes all of the C_i to evaluate to 1 under the rules of Boolean logic (i.e. the conjunction $C_1 \wedge C_2 \wedge \ldots \wedge C_k$ to evaluate to 1).

Theorem 1: LTE SU-MIMO PF-FDPS problem (i.e. maximization of the PF objective (1) under the only one MIMO mode constraint) is NP-hard.

Proof: We reduce 3-SAT to our problem, and our reduction will be based on viewing an instance of 3-SAT as a search over ways to choose a single term (to be satisfied) from each clause, subject to the constraint that one must not choose *conflicting* terms[1] from different clauses.

A decision version of our problem is to determine whether for a given frequency and spatial domain status S (i.e. a collection of value $\lambda^c_{i,j}$ across all users, RBs, and MIMO modes), there exists an allocation strategy that satisfies the only one MIMO mode constraint and results in an aggregate value at least w. We construct our problem instance in which the existence of a satisfiable allocation strategy for our problem depends on the existence of a satisfying truth assignment for a 3-SAT instance.

Consider an arbitrary instance of 3-SAT, with n variables $x_1, \ldots, x_n$ and k clauses $C_1, \ldots, C_k$. We first construct our

[1]We say that two terms *conflict* if one is equal to a variable x_i and the other is equal to its negation $\overline{x_i}$.

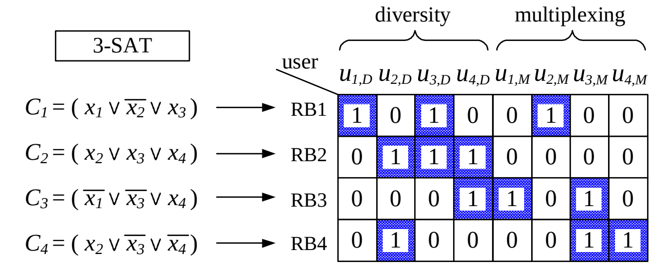

Fig. 1. The reduction from an example 3-SAT instance consisting of 4 variables and 4 clauses (i.e. $n = k = 4$). Dark-colored RBs represent the corresponding 3-SAT terms that appear in their matching clauses.

frequency and spatial domain status instance S as follows. A user in S corresponds to each variable in 3-SAT. More specifically, for each variable x_i and its negation $\overline{x_i}$, we have a user u_i for diversity mode $u_{i,D}$ and for multiplexing mode $u_{i,M}$, respectively. An RB in S matches with each clause in 3-SAT. Thus, we have n users and k RBs as shown in Figure 1. Since at most one user can be selected for each RB, this achieves our goal of choosing a single term in each clause that will evaluate to 1. However, we still need to explicitly encode the conflicting terms in a 3-SAT instance.

We now assign the scheduling metric values $\lambda^c_{i,j}$ in our frequency and spatial domain status S to model the conflicts of a given 3-SAT instance. In our problem formulation, either transmit diversity or spatial multiplexing mode can be selected for each user. It turns out that such one MIMO mode constraint is quite natural for encoding the conflicting property of 3-SAT. For each RB c, we set $\lambda^c_{i,D} = 1$ if x_i appears in clause Cc; similarly, we set $\lambda^c_{i,M} = 1$ if $\overline{x_i}$ appears in clause Cc, and $\lambda^c_{i,j} = 0$ otherwise. Finally, we define the target aggregate value $w = k$, which is the total number of clauses in a given 3-SAT instance. This completes the construction of the frequency and spatial domain status S.

We claim that our resulting construction S has a feasible allocation strategy with an aggregate value at least w if and only if the original 3-SAT instance is satisfiable. Indeed, if the 3-SAT is satisfiable, then each RB c in our domain S can be assigned to at least one user i with MIMO mode j whose metric value $\lambda^c_{i,j} = 1$. Let I be a set consisting of one such user $u_{i,j}$ for each RB. If two different MIMO modes were selected for a single user $u_{i,D}, u_{i,M} \in I$, then the corresponding terms in 3-SAT would conflict but this is not possible since they both evaluate to 1.

Conversely, suppose our domain S has a feasible allocation strategy OPT^* with a resulting aggregate value at least w. Then, first of all, the aggregate value is exactly w, and OPT^* must allocate one user i (with $\lambda^c_{i,j} = 1$) for each RB c. We now claim that there is a satisfying truth assignment ν for variables in the 3-SAT instance. For each variable x_i, if a user i is not allocated by OPT^*, then we arbitrarily set $\nu(x_i) = 1$. Otherwise, OPT^* selects exactly one MIMO mode for user i (i.e. $u_{i,D}$ or $u_{i,M}$) due to our problem constraint. If OPT^* chooses $u_{i,D}$, we set $\nu(x_i) = 1$, and otherwise we set $\nu(x_i) = 0$. By constructing ν in this way, all the clauses in the 3-SAT instance will evaluate to 1. ∎

B. Computational intractability in practice

Since we have proved in Theorem 1 that optimizing objective (1) is NP-hard, one might be tempted to optimize objective (1) by a “brute-force” search. Such an approach may work fine on relatively small-sized input using high computational power. That is, even though the problem itself is NP-hard, we may solve the problem by trying all possible user and MIMO mode combinations if the size of the typical instance is small. To examine whether or not brute-force search is practical, we first evaluate the running time of brute-force search for this problem.

Lemma 1: The running time of brute-force search for optimizing objective (1) under the one MIMO mode constraint is $O(n^n)$ if $m \geq n$, and $O(m^m)$ if $m < n$. (n users, m RBs) The proof is given in the Appendix.

In practical systems, both the number of users n and the number of RBs m can be quite large. For example, 3GPP LTE Release 8 supports a scalable bandwidth of 5, 10, 15, and 20 MHz (corresponding to 25, 50, 75, and 100 RBs, respectively) [3]. Furthermore, channel bandwidth up to 100 MHz (nearly 500 RBs) is being considered for LTE-Advanced. Moreover, we may have at least several tens of active users in a cell. Even in a sparse cell (say $n = 10$), it can take several seconds to complete the search (1 oper. $\approx$ 1 ns), which is too slow to schedule data every 1 ms in the real systems. Thus, we cannot optimize objective (1) in practice either.

C. Extension to MU-MIMO

Although we do not consider MU-MIMO in this paper, the hardness result can be naturally extended to the MU-MIMO.

Theorem 2: MU-MIMO FDPS problem (i.e. maximization of an objective function under the only one MIMO mode constraint) is NP-hard.

Proof: Since different users (under the user-paring constraint with respect to their precoding matrix) can be scheduled on different spatial streams over the same RB in MU-MIMO, the SU-MIMO is a special case of the MU-MIMO. This implies that MU-MIMO problem is no easier than our SU-MIMO problem that we have proved NP-hard in Theorem 1. ∎

III. Approximation algorithms

The hardness result in Section II provides a compelling reason to stop searching for an optimal solution algorithm for objective (1), and rather to pursue developing *approximation algorithms* that run in polynomial time and are able to find solutions that are guaranteed to be close to optimal.

In this section we present two approximation algorithms $Alg1$ and $Alg2$ that give constant-factor guaranteed performance bounds for objective (1). Interestingly, both algorithms provide $\frac{1}{2}$-approximations for objective (1), while $Alg2$ makes use of only a subset of CQI feedback information compared to $Alg1$ does.

Algorithm 1 : with full-CQI feedback

// Initial user-MIMO mode selection
for RB $c = 1$ to m **do**
  select the best user i as MIMO mode j with largest value $\lambda_{i,j}^c$
end for
// Conflict resolution
Let U be the set of users, each selected as both diversity and multiplexing MIMO modes on different RBs
while $U \neq \emptyset$ **do**
  pick a user $i \in U$
  Let K be the RBs assigned to user i
  if $\sum_{c \in K} \lambda_{i,D}^c \geq \sum_{c \in K} \lambda_{i,M}^c$ **then**
    assign all RBs $c \in K$ to user i as diversity mode
  else
    assign all RBs $c \in K$ to user i as multiplexing mode
  end if
  Set $U \leftarrow U - \{i\}$
end while

A. *Alg1: 1/2-Approximation with Full-CQI feedback*

We first present $Alg1$, a simple two-pass greedy algorithm similar to the one in [19] that utilizes full-CQI feedback as an input for decision (i.e. each user reports three CQIs per RB; one for diversity mode, the other two for dual-stream multiplexing MIMO mode, so that all the PF metric values $\lambda_{i,j}^c$ are known to the scheduler). The algorithm first makes one pass through the RBs for initial selection of user-MIMO mode; when it comes to RB c, it assigns c to the best user i as MIMO mode j whose PF metric value $\lambda_{i,j}^c$ is largest for RB c. This first-round procedure can select multiple MIMO modes for a single user on different RBs. Once the initial selection is done, the algorithm tries to resolve the MIMO mode conflict for the set of users i that are each selected as both diversity and multiplexing MIMO modes. Let K be the RBs assigned to one such user i in the initial selection. The algorithm compares the use of diversity mode for user i on those RBs $c \in K$ (i.e. the sum of PF metric values $\sum_{c \in K} \lambda_{i,D}^c$) and the multiplexing mode (i.e. the sum of values $\sum_{c \in K} \lambda_{i,M}^c$). Then, the user is forced to use the MIMO mode which gives a better aggregate value.

We now provide an analysis of algorithm $Alg1$.

Theorem 3: $Alg1$ is a $\frac{1}{2}$-approximation for objective (1).

Proof: Let T denote the aggregate PF value of the resulting assignment by $Alg1$. We here show that T is not much smaller than (more precisely, *at least* one half of) the maximum possible aggregate PF value T^* by the optimum algorithm, which we denote OPT^*.

For analysis on *upper bound*, we first relax the one MIMO mode constraint to allow multiple MIMO modes to be selected for a single user, then we can solve the resulting problem for objective (1) in polynomial time (i.e. by selecting the best user for each RB regardless of the MIMO mode restriction). This can violate our problem constraint but guarantees that its objective is *at least* the original problem optimum T^*:

$$\sum_c \Big(\max_{i,j} \lambda_{i,j}^c \Big) \geq T^*$$

We now consider the moment when $Alg1$ has just made the initial user-MIMO mode selection for all RBs, and let $K_{i,D}$ and $K_{i,M}$ be the sets of RBs assigned to user i as diversity mode and multiplexing mode, respectively. Since $Alg1$ always selects the best user-MIMO mode for each RB during the initial selection phase, we therefore have,

$$\begin{aligned} \sum_c \Big(\max_{i,j} \lambda_{i,j}^c \Big) &= \sum_i \sum_{c \in K_{i,D}} \lambda_{i,D}^c + \sum_i \sum_{c \in K_{i,M}} \lambda_{i,M}^c \\ &= \sum_i \Bigg(\sum_{c \in K_{i,D}} \lambda_{i,D}^c + \sum_{c \in K_{i,M}} \lambda_{i,M}^c \Bigg) \end{aligned} \tag{6}$$

Then, $Alg1$ performs the conflict resolution procedure to decide the resulting MIMO mode for one such user i, by the rule comparing the values summing over all the RBs assigned for user i; we define such a function $f(\cdot)$ by

$$f(i) = \arg\max_j \Big\{ \sum\nolimits_{c \in K_{i,D} \cup K_{i,M}} \lambda_{i,j}^c \Big\} \tag{7}$$

Thus, $f(i) = D$ if diversity mode is favored for user i, and $f(i) = M$ otherwise. We denote $\overline{f(i)}$ as complement of $f(i)$. So, (7) implies that,

$$\sum\nolimits_{c \in K_{i,D} \cup K_{i,M}} \lambda_{i,f(i)}^c \geq \sum\nolimits_{c \in K_{i,D} \cup K_{i,M}} \lambda_{i,\overline{f(i)}}^c \tag{8}$$

Equation (6) indicates that $\sum_{c \in K_{i,D}} \lambda_{i,D}^c + \sum_{c \in K_{i,M}} \lambda_{i,M}^c$ is an upper bound for a certain user i. By using Inequality (8), each term above can be bounded by:

$$\begin{aligned} &\sum_{c \in K_{i,D}} \lambda_{i,D}^c \\ &\leq \sum_{c \in K_{i,D}} \lambda_{i,D}^c + \sum_{c \in K_{i,M}} \lambda_{i,D}^c \\ &\leq \sum_{c \in K_{i,D}} \lambda_{i,f(i)}^c + \sum_{c \in K_{i,M}} \lambda_{i,f(i)}^c \quad \text{by Inequality (8)} \end{aligned} \tag{9}$$

Similarly, by applying Inequality (8) on the second term,

$$\sum_{c \in K_{i,M}} \lambda_{i,M}^c \leq \sum_{c \in K_{i,D}} \lambda_{i,f(i)}^c + \sum_{c \in K_{i,M}} \lambda_{i,f(i)}^c \tag{10}$$

Adding up two Inequalities (9) and (10), we get

$$\begin{aligned} \sum_{c \in K_{i,D}} \lambda_{i,D}^c + \sum_{c \in K_{i,M}} \lambda_{i,M}^c &\leq 2 \Bigg(\sum_{c \in K_{i,D}} \lambda_{i,f(i)}^c + \sum_{c \in K_{i,M}} \lambda_{i,f(i)}^c \Bigg) \\ &= 2 \sum\nolimits_{c \in K_{i,D} \cup K_{i,M}} \lambda_{i,f(i)}^c \end{aligned} \tag{11}$$

Note that in above Inequality (11), $\sum_{c \in K_{i,D} \cup K_{i,M}} \lambda_{i,f(i)}^c$ is the resulting aggregate value that $Alg1$ assigns for user i. Now we generalize Inequality (11) for every user i:

$$\begin{aligned} T^* &\le \sum_i \left(\sum_{c\in K_{i,D}} \lambda_{i,D}^c + \sum_{c\in K_{i,M}} \lambda_{i,M}^c \right) \\ &\le \sum_i \left(2 \sum\nolimits_{c\in K_{i,D}\cup K_{i,M}} \lambda_{i,f(i)}^c \right) \\ &= 2 \sum_i \sum\nolimits_{c\in K_{i,D}\cup K_{i,M}} \lambda_{i,f(i)}^c \\ &= 2T \end{aligned}$$

This immediately implies $T \ge \frac{1}{2}T^*$, which completes the proof. ∎

We now show that our analysis of algorithm $Alg1$ is essentially tight.

Theorem 4: For any constant $\varepsilon > 0$, there exists an instance on which $Alg1$ achieves at most a $1/(2-\varepsilon)$ fraction of the optimal value of objective (1).

Proof: It is not hard to give an example in which the solution by $Alg1$ is indeed close to a factor of $\frac{1}{2}$ away from optimum. The example is as follows. We have two users α and β, with two RBs in the system. The PF metric values are given by $\lambda_{\alpha,D}^1 = \lambda_{\alpha,M}^2 = 1$, and $\lambda_{\beta,D}^1 = 1-\varepsilon$. The rest are set to zero. The optimal algorithm assigns RB1 to user β and RB2 to user α as diversity mode and multiplexing MIMO mode, respectively. Hence the optimal value is $2-\varepsilon$. On the other hand, $Alg1$ assigns both RBs to user α (since $\lambda_{\alpha,D}^1 > \lambda_{\beta,D}^1$), with the resulting value to be 1, which is therefore at most a fraction $1/(2-\varepsilon)$ from the optimal. ∎

B. Alg2: 1/2-Approximation with Reduced-CQI feedback

While $Alg1$ with $\frac{1}{2}$-approximation seems a good candidate for the ideal case of full-CQI feedback assumption, it can make the signaling overhead quite large since each user needs to report CQIs for all the available MIMO modes. Moreover, this uplink CQI signaling overhead is a function of the number of RBs as well as that of active users in the system. Thus, in order to reduce the signaling overhead, we present another $\frac{1}{2}$-approximation algorithm $Alg2$, in which each user reports CQI only for the "better effective rate" MIMO mode per RB at a time, so that a significant amount of feedback signaling is reduced at the expense of the additional processing at the user side.[2] Now only half of the PF metric values $\lambda_{i,j}^c$ (i.e. for user i on RB c as better MIMO mode j; for a certain user i the better effective data rate $\hat{r}_{i,j}^c$ implies the better PF metric value $\lambda_{i,j}^c = \hat{r}_{i,j}^c / R_i$, since R_i is constant over all RBs) are known to the scheduler.

Similar to $Alg1$, the algorithm first makes one pass through the RBs for initial selection of user-MIMO mode among the reported ones; for RB c, it selects the best user i as MIMO mode j whose PF metric value $\lambda_{i,j}^c$ is largest. Once the initial selection is done, the algorithm handles a set of users assigned as multiple MIMO modes to enforce the MIMO

[2]In order for each user to determine the MIMO mode that results in better effective data rate $\hat{r}_{i,j}^c(t)$ for each RB, BLER tables are known/stored at the user terminal as well.

Algorithm 2 : with Reduced-CQI feedback

// Initial user-MIMO mode selection
for RB $c = 1$ to m **do**
  select the best user i as MIMO mode j with largest value $\lambda_{i,j}^c$ among the reported ones
end for
// Conflict resolution
Let U be the set of users, each selected as both diversity and multiplexing MIMO modes
while $U \neq \emptyset$ **do**
  pick a user $i \in U$
  Let K_D and K_M be the RBs assigned to user i as diversity mode and multiplexing mode, respectively
  if $\sum_{c\in K_D} \lambda_{i,D}^c \ge \sum_{c\in K_M} \lambda_{i,M}^c$ **then**
    all RBs $c \in K_D \cup K_M$ to user i as diversity mode
    (for RBs $c \in K_M$, the lowest data rate is selected)
  else
    all RBs $c \in K_D \cup K_M$ to user i as multiplexing mode
    (for RBs $c \in K_D$, the lowest data rate is selected)
  end if
  Set $U \leftarrow U - \{i\}$
end while

mode constraint. Let K_D and K_M be the RBs assigned to user i as diversity mode and multiplexing mode respectively. The algorithm compares the diversity mode for user i on RBs $c \in K_D$ (i.e. the sum of PF metric values $\sum_{c\in K_D} \lambda_{i,D}^c$) and the multiplexing mode on RBs $c \in K_M$ (i.e. the sum of values $\sum_{c\in K_M} \lambda_{i,M}^c$). Then, the user is forced to use the MIMO mode which gives a better aggregate value, and is assigned on RBs $c \in K_D \cup K_M$. Note that $Alg2$ cannot compare $\sum_{c\in K_D\cup K_M} \lambda_{i,D}^c$ and $\sum_{c\in K_D\cup K_M} \lambda_{i,M}^c$ as in $Alg1$, since $\sum_{c\in K_M} \lambda_{i,D}^c$ and $\sum_{c\in K_D} \lambda_{i,M}^c$ are not known to the scheduler in $Alg2$. Likewise, once user i is chosen to use MIMO mode j on RBs $c \in K_j \cup K_{j'}$, then the lowest data rate is selected on RBc $c \in K_{j'}$ as a conservative approach (increased reliability) since CQIs for MIMO mode j on RBs $c \in K_{j'}$ are not reported by user i.

We now provide an analysis of algorithm $Alg2$.

Theorem 5: $Alg2$ is a $\frac{1}{2}$-approximation for objective (1).

Proof: The proof is quite similar to the analysis of $Alg1$. Here, we let T be the resulting aggregate PF value by $Alg2$. As before, we first use the relaxation of the one MIMO mode constraint to obtain an upper bound on the optimum T^*:

$$\sum_c \left(\max_{i,j} \lambda_{i,j}^c \right) \ge T^*$$

At the moment when $Alg2$ has just made the initial user-MIMO mode selection for all RBs, we let $K_{i,D}$ and $K_{i,M}$ be the sets of RBs assigned to user i as diversity mode and multiplexing mode, respectively. Note that the resulting initial selection by $Alg2$ is the same as that by $Alg1$ since each user reports CQI for a better MIMO mode per RB (i.e. better effective data rate) in $Alg2$. Therefore, during the initial selection phase, $Alg2$ always selects the best user-MIMO mode for each RB:

$$\sum_c \Big(\max_{i,j} \lambda_{i,j}^c\Big) = \sum_i \Bigg(\sum_{c\in K_{i,D}} \lambda_{i,D}^c + \sum_{c\in K_{i,M}} \lambda_{i,M}^c\Bigg) \quad (12)$$

Then, $Alg2$ performs the conflict resolution procedure to decide the resulting MIMO mode for one such user i, by the rule comparing the values summing over only the diversity RBs $\in K_{i,D}$ and only the multiplexing RBs $\in K_{i,M}$ assigned for user i separately; we define such a function $g(\cdot)$ by

$$g(i) = \arg\max_j \Big\{\sum\nolimits_{c\in K_{i,j}} \lambda_{i,j}^c\Big\} \quad (13)$$

Thus, $g(i) = D$ if diversity mode is favored for user i, and $g(i) = M$ otherwise. We denote $\overline{g(i)}$ as complement of $g(i)$. So, (13) implies that,

$$\sum\nolimits_{c\in K_{i,g(i)}} \lambda_{i,g(i)}^c \geq \sum\nolimits_{c\in K_{i,\overline{g(i)}}} \lambda_{i,\overline{g(i)}}^c \quad (14)$$

Equation (12) indicates that $\sum_{c\in K_{i,D}} \lambda_{i,D}^c + \sum_{c\in K_{i,M}} \lambda_{i,M}^c$ is an upper bound for a certain user i. By using Inequality (14), each term above can be bounded by:

$$\sum\nolimits_{c\in K_{i,D}} \lambda_{i,D}^c \leq \sum\nolimits_{c\in K_{i,g(i)}} \lambda_{i,g(i)}^c \quad (15)$$

$$\sum\nolimits_{c\in K_{i,M}} \lambda_{i,M}^c \leq \sum\nolimits_{c\in K_{i,g(i)}} \lambda_{i,g(i)}^c \quad (16)$$

Adding up two Inequalities (15) and (16), we get

$$\sum_{c\in K_{i,D}} \lambda_{i,D}^c + \sum_{c\in K_{i,M}} \lambda_{i,M}^c \leq\ 2 \sum_{c\in K_{i,g(i)}} \lambda_{i,g(i)}^c \quad (17)$$

Note that in above Inequality (17), $\sum_{c\in K_{i,g(i)}} \lambda_{i,g(i)}^c$ is the resulting aggregate value (more precisely, the actual value can be larger, since $Alg2$ assigns *at least* the lowest data rate for the unreported CQIs.) that $Alg2$ assigns for user i. Now we generalize Inequality (17) for every user i:

$$\begin{aligned} T^* &\leq \sum_i \Bigg(\sum_{c\in K_{i,D}} \lambda_{i,D}^c + \sum_{c\in K_{i,M}} \lambda_{i,M}^c\Bigg) \\ &\leq \sum_i \Big(2 \sum\nolimits_{c\in K_{i,g(i)}} \lambda_{i,g(i)}^c\Big) \\ &= 2 \sum_i \sum\nolimits_{c\in K_{i,g(i)}} \lambda_{i,g(i)}^c \\ &\leq 2T \end{aligned}$$

This immediately implies $T \geq \frac{1}{2}T^*$, which completes the proof. ■

We conclude this section by showing that our analysis of algorithm $Alg2$ is tight.

Theorem 6: For any constant $\varepsilon > 0$, there exists an instance on which $Alg2$ achieves at most a $1/(2-\varepsilon)$ fraction of the optimal value of objective (1).

Proof: Here we use a much simpler example than the one in Theorem 4. The example is as follows. We have only one user α, with two RBs in the system. The PF metric values are given by $\lambda_{\alpha,D}^1 = 1$, $\lambda_{\alpha,D}^2 = 0$, and $\lambda_{\alpha,M}^1 = \lambda_{\alpha,M}^2 = 1 - \frac{1}{2}\varepsilon$. The optimal algorithm, as well as $Alg1$, assigns both RBs to user α as multiplexing MIMO mode. Hence the optimal value is $2-\varepsilon$. On the other hand, $Alg2$ assigns both RBs to user α as diversity mode (since $\lambda_{\alpha,D}^1 > \lambda_{\alpha,M}^2$), with the resulting value to be 1, which is therefore at most a $1/(2-\varepsilon)$ fraction of the optimal value. ■

Remark. The performance of $Alg2$ can be further improved by a more advanced assignment strategy for the RBs $c \in K_{j'}$ that user i is assigned as MIMO mode j without CQIs for mode j; for example, instead of using the conservative approach, it may try to select another user i' who (i) has the best possible CQI for MIMO mode j reported on RBs $c \in K_{j'}$ and (ii) does not violate the one MIMO mode constraint over RBs that i' is already, if any, assigned. Such a strategy may, however, incur another round of iterative optimization search process.

IV. SIMULATIONS

In order to evaluate the performance of the proposed algorithms, we conducted MIMO-OFDMA system level simulations based on 3GPP LTE system model. We used trace files generated as specified in 3GPP deployment evaluation [2], based on the Typical Urban channel model. Table I summarizes a list of the default simulation parameters and assumptions.

We analyze the performance of the algorithms in terms of cell throughput, short-term fairness, as well as uplink CQI signaling overhead and assess how well they emulate the PF criteria in the MIMO-FDPS setting. We use Jain's fairness index [11], measured by the data-rate fairness criterion:

$$F_\phi(\Delta t) = \frac{[\sum_{i=1}^N \phi_i(\Delta t)]^2}{[N \cdot \sum_{i=1}^N \phi_i(\Delta t)^2]},$$

where $\phi_i(\Delta t)$ denotes the actual data-rate user i achieved in time interval Δt, with N users in the system; $F_\phi(\Delta t) = 1$ implies that all users received equal data-rate within time Δt.

We first measure the system throughput of our algorithms, as well as 1×2 SIMO FDPS-only as a reference with varying the number of active users in the cell. As shown in Figure 2, both

TABLE I Simulation parameters

Parameter	Setting
System bandwidth	20 MHz
Subcarriers per RB	12
RB bandwidth	180 kHz
Number of RBs	96
Cell-level user distribution	Uniform
Number of active users in cell	10, 20, 30, 40, 50
Traffic model	Infinitely backlogged
Transmission time interval (TTI)	1 ms
Channel model	Typical Urban
User speed	3, 30, 120 km/h
User receiver	2x2/MMSE/ZF
Modulation/coding rate settings	QPSK: 1/4 - 8/9 16QAM: 1/2 - 8/9 64QAM: 2/3 - 8/9
HARQ model	Ideal chase combining
HARQ Aak/Nack delay	8 ms
Max. number of HARQ retransmission	3

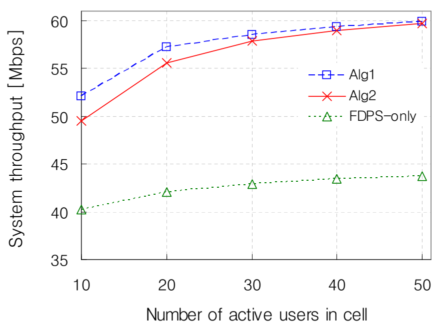

Fig. 2 System throughput vs. number of active users

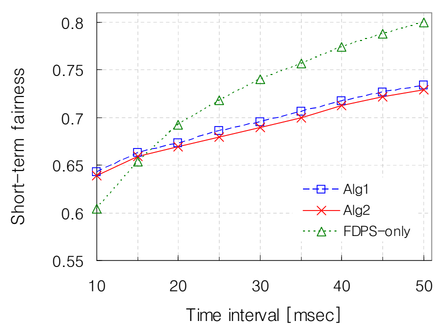

Fig. 3 Short-term fairness (30 users) vs. time interval

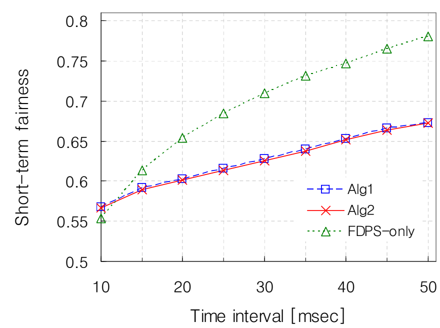

Fig. 4 Short-term fairness (50 users) vs. time interval

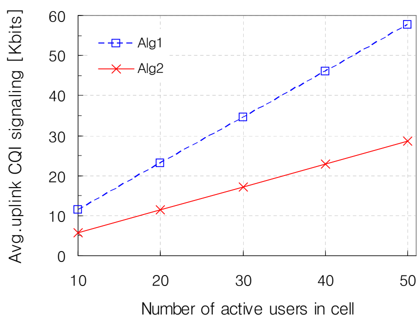

Fig. 5 Avg. uplink CQI signaling (Kbits) per CQI update

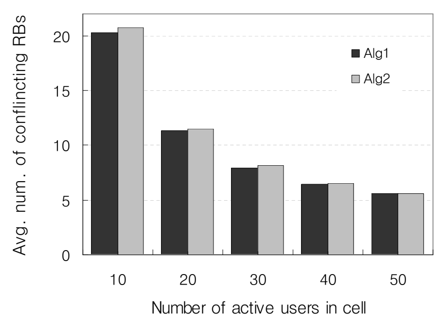

Fig. 6 Avg. number of conflicting RBs per user with multiple MIMO modes in 1 TTI

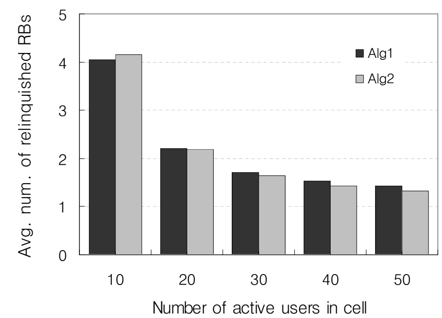

Fig. 7 Avg. num. of relinquished RBs per user with multiple MIMO modes in 1 TTI

MIMO-FDPS algorithms $Alg1$ and $Alg2$ offer considerable gains in the order of 24-35% over the reference case of 1×2 SIMO FDPS-only. We also observe that $Alg1$ consistently provides a slightly better throughput performance than $Alg2$. Figure 3 and 4 present the short-term data-rate fairness $F_\phi(\Delta t)$ in the cell of 30 and 50 active users, respectively by varying the time interval window Δt from 10 ms (i.e. 10 TTI) to 50 ms. Based on Figure 3 and 4, we observe that both SU-MIMO-FDPS algorithms $Alg1$ and $Alg2$ have similar short-term fairness. However, the short term fairness of SU-MIMO-FDPS algorithms is worse compared to FDPS-only algorithm. This result seems quite intuitive in the sense that SU-MIMO essentially facilitates the peak user data rate improvement (i.e. only one user can be scheduled each RB with an opportunity of being decided as dual-stream transmission), while FDPS-only tends to promote the average data rate enhancement that helps to further improve the fairness among users.

We now estimate the uplink CQI feedback signaling overhead. The resolution for CQI has been decided to be 4 bits per RB per spatial stream in 3GPP LTE Release 8 [3]. In the simulations, the system bandwidth of 20 MHz is divided into 96 RBs (each with 180 kHz). For $Alg1$ with the full-CQI feedback assumption, since the CQI is reported for both single-stream diversity mode and dual-stream multiplexing mode (one CQI per stream), the CQI signaling requires 4 × 96 × 3 = 1152 bits/update/user. On the other hand, $Alg2$ needs the CQI only for the better MIMO mode so that the CQI overhead can be reduced to 4 or 8 bits (instead of 12 bits) per RB, depending on which mode is favored at each update. Figure 5 shows the simulation result of the average CQI signaling overhead per update with varying the number of active users. $Alg2$ reduces the CQI signaling overhead by around 50% over the signaling requirement of $Alg1$.[3]

A closer inspection of Figure 2 reveals that in spite of the full-CQI utilization, $Alg1$ displays only marginal throughput gain over $Alg2$. With increasing number of active users, the throughput gain of $Alg1$ over $Alg2$ gets smaller (e.g. when $n = 50$, its gain is within 1% over $Alg2$). To understand why $Alg2$ shows similar performance to $Alg1$ with a large number of users in the cell, we record the number of the RBs assigned to a user with multiple MIMO modes selected by the initial user-MIMO mode selection procedure, and refer to such RBs as *conflicting* RBs. In addition, we trace the number of RBs in which the conflict resolution procedure forces a user with conflicting RBs to relinquish the originally selected MIMO mode, and refer to them as *relinquished* RBs. Figure 6 and 7 plot the average number of conflicting RBs and relinquished RBs per user with multiple MIMO mode selected in 1 TTI, respectively. It is observed that both conflicting RBs

[3]The feedback reduction is still an open issue yet a key feature in LTE. Further feedback compression techniques such as wavelet based signal analysis and long-term based signaling are currently under discussion in 3GPP.

and relinquished RBs decrease with the number of active users. This implies that the *multiuser diversity* might help reduce the probability of conflicting assignment during the initial selection procedure of the approximation algorithms. Hence, a large number of users results in similar performance for both algorithms.

Now we recall that our ultimate goal is to maximize the PF criteria (i.e. maximizing $\sum_i \log R_i$, where R_i is the long-term service rate for user i) in the MIMO-FDPS context. We assess how well our approximation algorithms emulate the proportional fair objective in this problem framework. Table II shows the values of the PF criteria (i.e. $\sum_i \log R_i$) with the number of active users (10, 20, 30, 40, and 50 users) in the cell. In all cases we observe that $Alg2$ has the values

TABLE II PF criteria ($\sum_i \log R_i$) of $Alg1$ and $Alg2$

$\sum_i \log R_i$	10	20	30	40	50
Alg1	85.7	159.3	227.4	292.2	354.6
Alg2	85.2	158.8	227.1	292.0	354.5

of $\sum_i \log R_i$ very close (within only a small factor) to that of $Alg1$. Furthermore, a closer look at the table shows that the gap between the values of $Alg1$ and $Alg2$ gets smaller with increasing number of active users. This trend indeed conforms to our earlier results on the cell throughput and short-term fairness of our algorithms (recall that the PF criterion is a comprehensive metric that takes both throughput and fairness into account). Therefore, $Alg2$ maintains acceptable performance (i.e. only marginal performance loss over $Alg1$) along with CQI signaling reduction of about 50% over $Alg1$ with full-CQI feedback requirement.

V. Conclusions

In this paper we consider the LTE SU-MIMO FDPS scheduling problem of maximizing PF objective under the constraint that only one MIMO mode per single user can be used for each time instance. We first prove that SU-MIMO FDPS is NP-hard under the LTE requirement. We then present two SU-MIMO FDPS approximation algorithms $Alg1$ and $Alg2$. While both give $\frac{1}{2}$-approximations for the objective, $Alg2$ makes use of only a subset of CQI feedback information compared to $Alg1$.

Based on 3GPP LTE framework, simulation results reveal that the approximation algorithms $Alg1$ and $Alg2$ offer measurable gains in the order of 24-35% over the 1×2 SIMO FDPS-only (reference case). Moreover, $Alg2$ achieves a CQI signaling reduction of 50% with only 1-5% performance degradation over $Alg1$ that requires the full-CQI feedback. We also prove that the LTE MU-MIMO FDPS problem is NP-hard. Future work would entail extending the results presented in this paper to develop efficient algorithms for MU-MIMO FDPS scheduling.

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Appendix

A. Proof of Lemma 1

Proof: We first consider the number of possible combination of users to be scheduled (under only one MIMO mode constraint) in one time slot where $m \geq n$ (i.e. the number of RBs is greater than that of users): $\binom{n}{1} \cdot 2 + \binom{n}{2} \cdot 2^2 \cdots + \binom{n}{n} \cdot 2^n$. Since we have m RBs, the total search space is:

$$T(n,m) = \sum_{i=1}^{n} \binom{n}{i} \cdot 2^i \cdot i^m \geq \sum_{i=1}^{n} \binom{n}{i} \cdot 2^i \cdot i^n = O(n^n)$$

In the case when $m < n$, we cannot assign more than m users at a time: $\binom{n}{1} \cdot 2 + \binom{n}{2} \cdot 2^2 \cdots + \binom{n}{m} \cdot 2^m$. Then the total search space is: $T(n,m) = \sum_{i=1}^{m} \binom{n}{i} \cdot 2^i \cdot i^m = O(m^m)$ ■